SAFE™

Integrated
Analysis and Design of Slabs
by
The Finite Element Method

VERIFICATION MANUAL



Computers and Structures, Inc.
Berkeley, California, USA

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THE USER MUST EXPLICITLY UNDERSTAND THE ASSUMPTIONS OF THE PROGRAM AND MUST INDEPENDENTLY VERIFY THE RESULTS.

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Introduction

Overview

SAFE is a reinforced concrete slab and basemat analysis and design program based on the finite element method. This manual contains a collection of examples verifying the accuracy and applicability of the program.

The first seven examples verify the accuracy of the elements and the solution algorithms used in SAFE. These examples compare displacements and member internal forces predicted by SAFE with known theoretical solutions for various slab support and load conditions.

The next seven examples verify the applicability of SAFE in calculating design moments in slabs by comparing results for practical slab geometries with experimental results and/or results using ACI 318-95 recommendations.

The last three examples verify the design algorithms used in SAFE for flexural and shear design, using the ACI 318-95 recommendations, by comparing SAFE results with hand calculations.

Organization of Manual

This manual includes one chapter for each example. A typical chapter consists of the following sections:

Description: A brief description of the problem is given.

Data: Geometry, material properties, load cases, boundary conditions, etc. are defined.

Modeling Procedure: Highlights of adopted modeling approach and special features of the model are provided.

Comparison of Results: SAFE results are compared with those of theoretical and/or experimental solution.

File Reference: This section contains referenced filenames associated with individual examples. These files are provided with the complete SAFE package so that the user can independently cross reference and validate data reported in this manual.

Example 1

Simply-Supported Rectangular Plate

Description

A simply supported rectangular plate, as shown in Figure 1-1, is analyzed for three different load conditions — uniformly distributed load over the slab (UL), a concentrated point load at the center of the slab (PL), and a line load along a centerline of the slab (LL). Closed form solutions to these problems are given in Timoshenko and Woinowsky (1959) employing a double Fourier Series (Navier's solution) or a single series (Lévy's solution). The numerically computed deflections, local moments, average strip moments, local shears, and average strip shears obtained from SAFE are compared with the corresponding closed form solutions.

Data

Plate size,	$a \times b$	= 360 in × 240 in
Plate thickness,	T	= 8 inches
Modulus of elasticity,	E	= 3000 ksi
Poisson's ratio,	ν	= 0.3

Load Cases:

(UL)	Uniform load,	q	= 100	psf
(PL)	Point load,	P	= 20	kips
(LL)	Line load,	q_l	= 1	kip/ft

Modeling Procedure

To test convergence, the problem is analyzed employing three mesh sizes, 4×4, 8×8, and 12×12, as shown in Figure 1-2. The slab is modeled using thin plate elements in SAFE. The simply supported edges are modeled as line supports with a large vertical stiffness. Three load cases are considered. Self-weight is not included in these analyses.

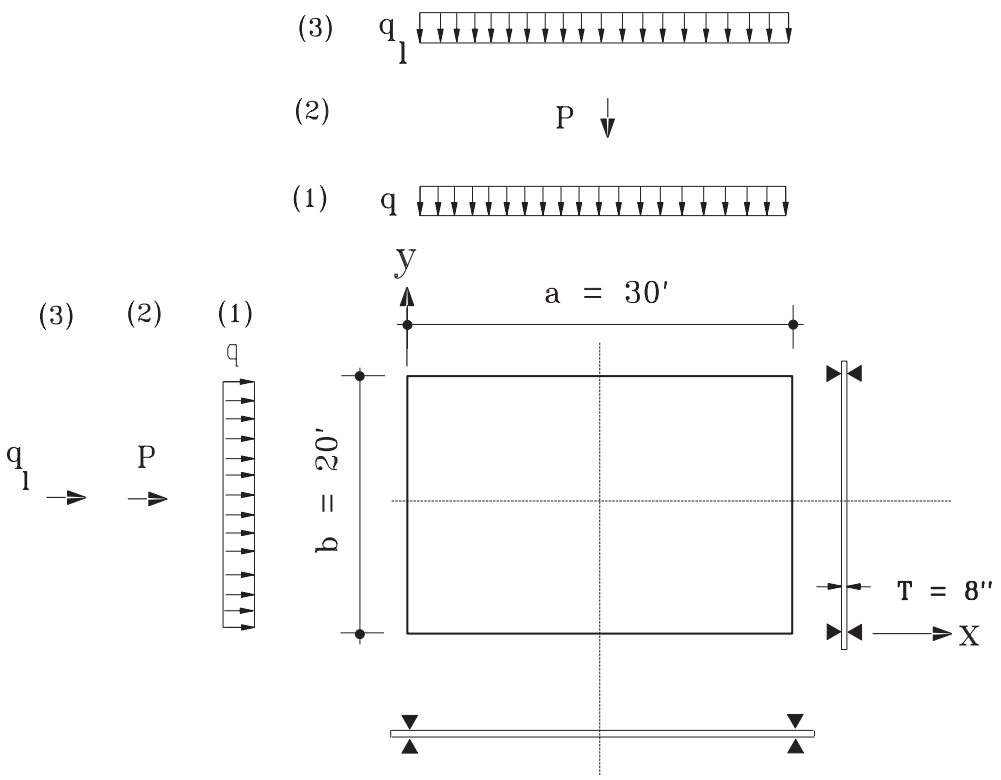
To obtain design moments, the plate is divided into three strips — two edge strips and one middle strip — each way, based on the ACI 318-95 definition of design strip widths for a two-way slab system as shown in Figure 1-3. For comparison with the theoretical results, load factors of unity are used and each load case is processed as a separate load combination.

Comparison of Results

The deflections of four different points for three different mesh refinements are shown for the three load cases in Table 1-1. The theoretical solutions based on Navier's formulations are also shown for comparison. This figure also shows the convergence of the SAFE solution with mesh refinement. It can be observed from Table 1-1 that the deflection obtained from SAFE converges monotonically to the theoretical solution with mesh refinement. Moreover, the agreement is good for even the coarse mesh (4×4). The numerically obtained local-moments and local-shears at critical points are compared with that of the theoretical values in Table 1-2 and Table 1-3, respectively. Close agreement has been observed. A noticeable discrepancy, however, occurs for the case of the point load (load case PL) in the region close to the application of the point load, where the theoretical model has a singularity. The average strip-moments for the load cases are compared with the theoretical average strip-moments in Table 1-4, where excellent agreement can be observed. It should be noted that in calculating the theoretical solution, a sufficient number of terms from the series are taken into account to achieve the accuracy of the theoretical solutions.

File Reference

The files for the 4×4, 8×8, and 12×12 meshes are S01a.FDB, S01b.FDB, and S01c.FDB, respectively. These files are included in the SAFE package.



Material Properties :

Modulus of Elasticity = 3000 ksi
Poisson's Ratio = 0.3

Load Cases :

(1) Uniform Load, q = 100 psf
(2) Point Load, P = 20 kips
(3) Line Load, q_l = 1 kip/ft

Figure 1-1
Rectangular Plate Example 1

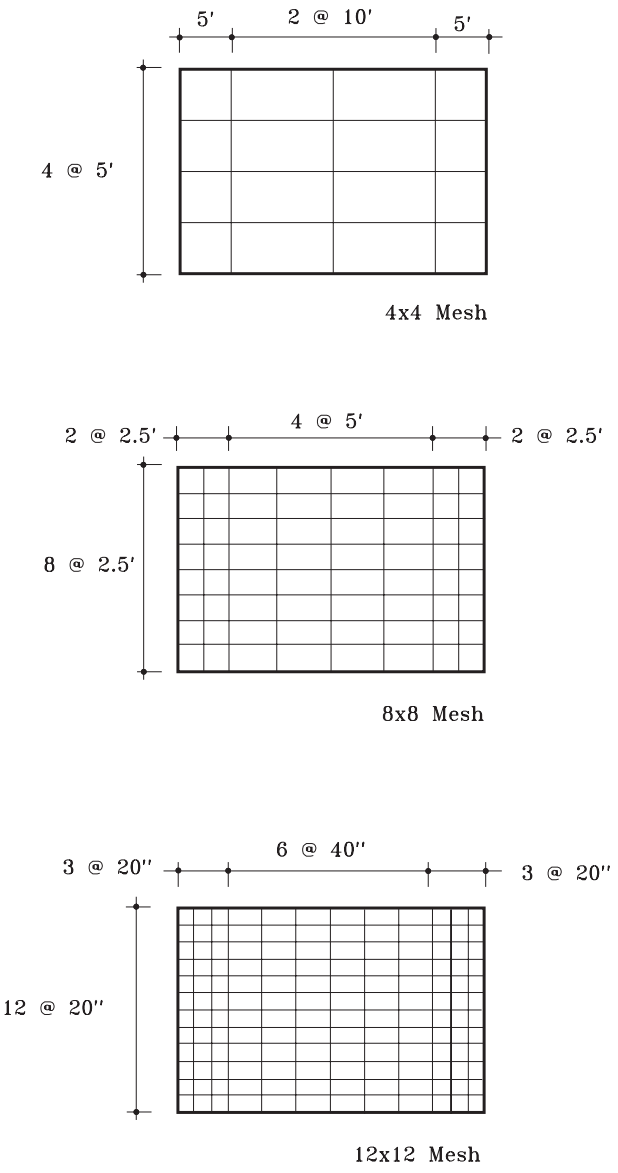


Figure 1-2
SAFE Meshes for Rectangular Plate

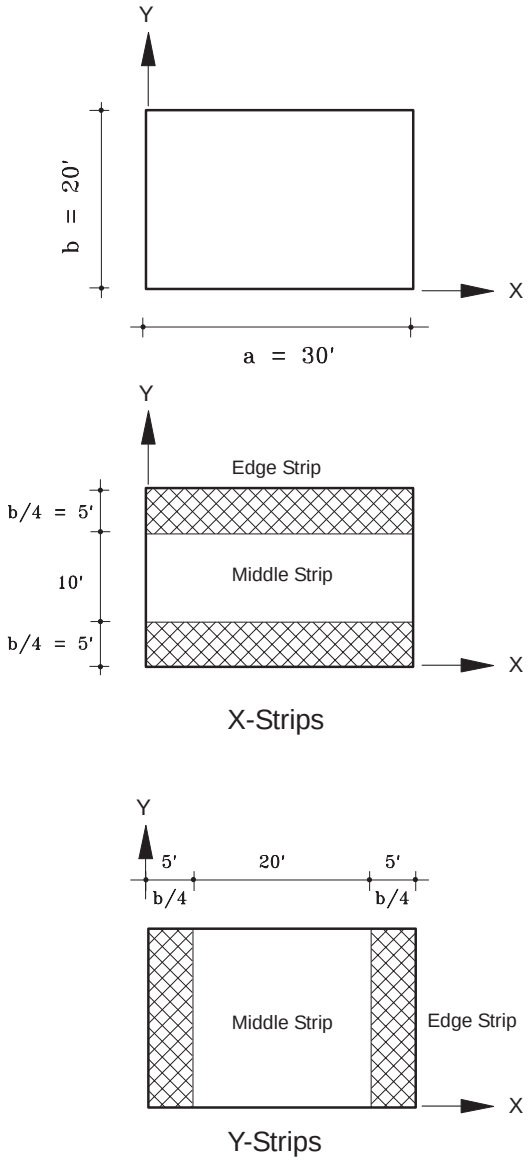


Figure 1-3
SAFE Definition of Design Strips

Load Case	Location		SAFE Displacement (in)			Theoretical Displacement (in)
	X (in)	Y (in)	4x4 Mesh	8x8 Mesh	12x12 Mesh	
UL	60	60	0.0491	0.0492	0.0493	0.0492961
	60	120	0.0685	0.0684	0.0684	0.0684443
	180	60	0.0912	0.0908	0.0907	0.0906034
	180	120	0.1279	0.1270	0.1267	0.1265195
PL	60	60	0.0371	0.0331	0.0325	0.0320818
	60	120	0.0510	0.0469	0.0463	0.0458716
	180	60	0.0914	0.0829	0.0812	0.0800715
	180	120	0.1412	0.1309	0.1283	0.1255747
LL	60	60	0.0389	0.0375	0.0373	0.0370825
	60	120	0.0593	0.0570	0.0566	0.0562849
	180	60	0.0735	0.0702	0.0696	0.0691282
	180	120	0.1089	0.1041	0.1032	0.1024610

Table 1-1
Comparison of Displacements

Load Case	Location		Moments (kip-in/in)					
			M_x		M_y		M_{xy}	
	X (in)	Y (in)	SAFE (8x8)	Analytical (Navier)	SAFE (8x8)	Analytical (Navier)	SAFE (8x8)	Analytical (Navier)
UL	150	15	0.42	0.45	0.73	0.81	0.31	0.30
	150	45	1.16	1.18	1.95	2.02	0.26	0.26
	150	75	1.66	1.69	2.69	2.77	0.17	0.17
	150	105	1.92	1.95	3.04	3.12	0.06	0.06
PL	150	15	0.37	0.37	0.36	0.36	0.44	0.47
	150	45	1.11	1.13	1.13	1.14	0.48	0.51
	150	75	1.92	1.90	2.16	2.20	0.56	0.59
	150	105	2.81	2.41	3.85	3.75	0.42	0.47
LL	150	15	0.26	0.26	0.34	0.34	0.24	0.24
	150	45	0.77	0.77	1.06	1.08	0.21	0.20
	150	75	1.25	1.25	1.91	1.92	0.14	0.14
	150	105	1.69	1.68	2.94	2.95	0.05	0.05

Table 1-2
Comparison of Local Moments

Load Case	Location		Shears ($\times 10^{-3}$ kip/in)			
			V_x		V_y	
	X (in)	Y (in)	SAFE (8x8)	Analytical (Navier)	SAFE (8x8)	Analytical (Navier)
UL	15	45	27.5	35.2	5.8	7.6
	45	45	16.1	21.2	17.2	21.0
	90	45	7.3	10.5	28.4	33.4
	150	45	1.7	3.0	36.2	40.7
PL	15	45	4.8	8.7	2.4	2.6
	45	45	6.7	9.8	8.6	8.3
	90	45	12.5	13.1	20.5	19.2
	150	45	11.2	11.2	34.8	43.0
LL	15	45	13.2	15.7	4.6	5.7
	45	45	10.9	13.0	13.5	16.2
	90	45	5.8	7.6	22.6	26.5
	150	45	1.4	2.2	29.0	32.4

Table 1-3
Comparison of Local Shears

Load Case	Moment Direction	Strip	SAFE Average Strip Moments (kip-in/in)			Theoretical Average Strip Moments (kip-in/in)
			4x4 Mesh	8x8 Mesh	12x12 Mesh	
UL	$\overline{M}_x$ $x = 180''$	Column	0.758	0.800	0.805	0.810
		Middle	1.843	1.819	1.819	1.820
	$\overline{M}_y$ $y = 120''$	Column	0.975	0.989	0.992	0.994
		Middle	2.703	2.770	2.782	2.792
PL	$\overline{M}_x$ $x = 180''$	Column	0.993	0.960	0.927	0.901
		Middle	3.334	3.857	3.975	3.950
	$\overline{M}_y$ $y = 120''$	Column	0.440	0.548	0.546	0.548
		Middle	3.521	3.371	3.358	3.307
LL	$\overline{M}_x$ $x = 180''$	Column	0.548	0.527	0.522	0.519
		Middle	1.561	1.491	1.482	1.475
	$\overline{M}_y$ $y = 120''$	Column	1.208	1.380	1.424	1.432
		Middle	3.083	3.200	3.221	3.200

Table 1-4
Comparison of Average Strip Moments

Example 2

Rectangular Plate with Built-In Edges

Description

A fully clamped rectangular plate, as shown in Figure 2-1, is analyzed for three different load conditions. A theoretical solution to this problem, employing a single series (Lévy's solution) is given in Timoshenko and Woinowsky (1959). The numerically computed deflections obtained from SAFE are compared with the theoretical values.

This example is similar to Example 1 in terms of geometric descriptions and material properties. The load cases are also the same as that of Example 1. However, the boundary conditions are different. All edges are fixed as are shown in Figure 2-1. The finite element meshes used to model this problem are shown in Figure 1-2. The numerical data for this problem are given in the following section.

Data

Plate size,	$a \times b$	$= 360'' \times 240''$
Plate thickness,	T	= 8 inches
Modulus of elasticity,	E	= 3000 ksi
Poisson's ratio,	ν	= 0.3

Load Cases:

(UL)	Uniform load,	q	= 100	psf
(PL)	Point load,	P	= 20	kips
(LL)	Line load,	q_l	= 1	kip/ft

Modeling Procedure

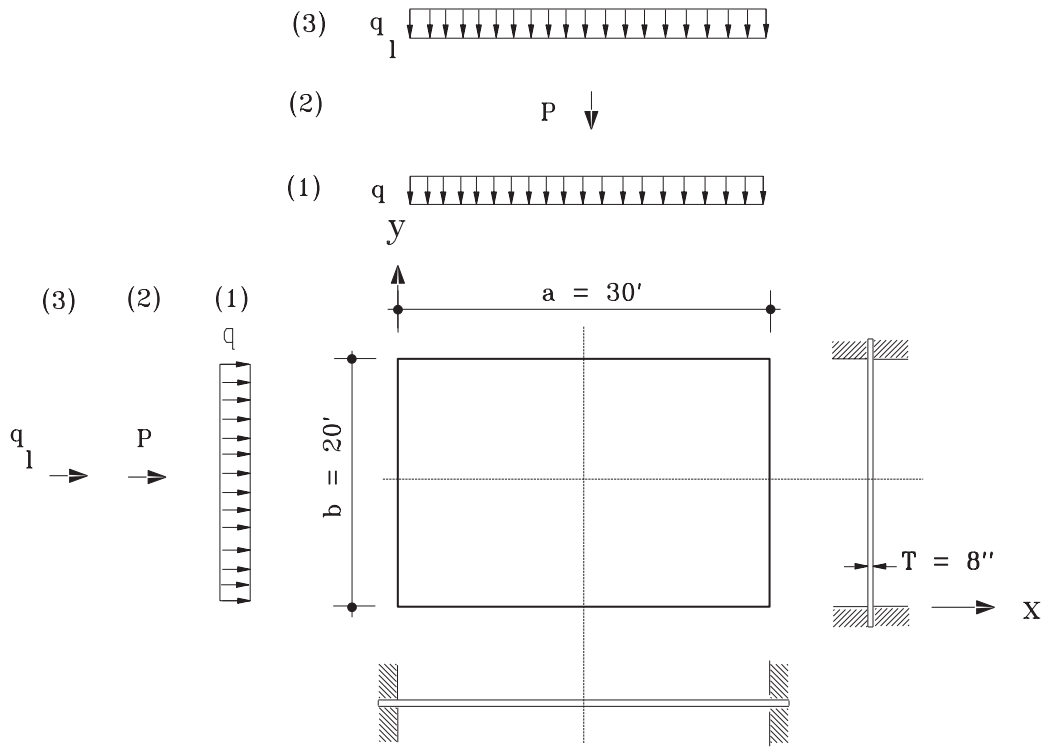
To test convergence, the problem is analyzed using three mesh sizes, 4×4, 8×8, and 12×12, as shown in Figure 1-2. The plate is modeled using thin plate elements available in SAFE. The fixed edges are modeled as line supports with large vertical and rotational stiffnesses. The self-weight of the plate is not included in any of the load cases.

Comparison of Results

The numerical displacements obtained from SAFE are compared with those obtained from the theoretical solution in Table 2-1. The theoretical results are based on tabular values given in Timoshenko and Woinowsky (1959). A comparison of deflections for the three load cases shows a fast convergence to the theoretical values with successive mesh refinement.

File Reference

The files for the 4×4, 8×8, and 12×12 meshes are S02a.FDB, S02b.FDB, and S02c.FDB, respectively. These files are included in the SAFE package.



Material Properties :

Modulus of Elasticity = 3000 ksi
Poisson's Ratio = 0.3

Load Case No. :

(1) Uniform Load, q = 100 psf
(2) Point Load, p = 20 k
(3) Line Load, q_l = 1 kip/ft

Figure 2-1
Rectangular Plate with All Edges Clamped

Load Case	Location		SAFE Displacement (in)			Theoretical Displacement (in)
	X (in)	Y (in)	4x4 Mesh	8x8 Mesh	12x12 Mesh	
UL	60	60	0.0098	0.0090	0.0089	
	60	120	0.0168	0.0153	0.0150	
	180	60	0.0237	0.0215	0.0210	
	180	120	0.0413	0.0374	0.0366	0.036036
PL	60	60	0.0065	0.0053	0.0052	
	60	120	0.0111	0.0100	0.0100	
	180	60	0.0315	0.0281	0.0272	
	180	120	0.0659	0.0616	0.0598	0.057453
LL	60	60	0.0079	0.0072	0.0071	
	60	120	0.0177	0.0161	0.0158	
	180	60	0.0209	0.0188	0.0184	
	180	120	0.0413	0.0375	0.0367	

Table 2-1
Comparison of Displacements

Example 3

Rectangular Plate with Mixed Boundary

Description

The plate, shown in Figure 3-1, is analyzed for uniform load only. The edges along $x = 0$ and $x = a$ are simply supported, the edge along $y = b$ is free, and the edge along $y = 0$ is fully fixed. An explicit analytical expression for the deflected surface is given in Timoshenko and Woinowsky (1959). The deflections obtained from SAFE are compared with the theoretical values.

The geometrical description and material properties of this problem are the same as that of the Example 1. However, the boundary conditions differ. The detailed problem is described in the following section and shown in Figure 3-1. The gridlines used to generate the finite element model are shown in Figure 1-2. In this example only one load case is considered.

Data

Plate size,	$a \times b$	= $360'' \times 240''$
Plate thickness,	T	= 8 inches
Modulus of elasticity,	E	= 3000 ksi
Poisson's ratio,	ν	= 0.3

Load Cases:

Uniform load,	q	= 100 psf

Modeling Procedure

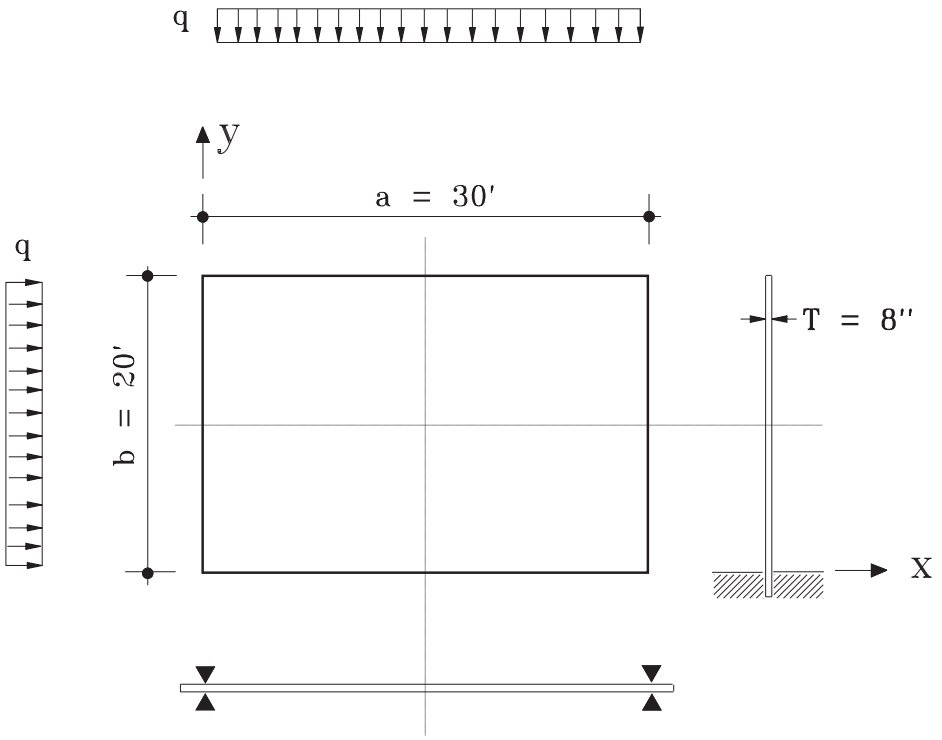
To test convergence, the problem is analyzed employing three mesh sizes, 4×4, 8×8, and 12×12, as shown in Figure 1-2. The plate is modeled using thin plate elements available in SAFE. The two simply supported edges are modeled as line supports with large vertical stiffnesses. The fixed edge is modeled as a line support with large vertical and rotational stiffnesses. The self-weight of the plate is not included in the analysis.

Comparison of Results

The numerical solution obtained from SAFE is compared with the theoretical solution which is given by Lévy (Timoshenko and Woinowsky, 1959). Comparison of deflections shows monotonic convergence to the theoretical values with successive mesh refinement as depicted in Table 3-1. It is to be noted that even with a coarse mesh (4×4) the agreement is very good.

File Reference

The files for the 4×4, 8×8, and 12×12 meshes are S03a.FDB, S03b.FDB, and S03c.FDB, respectively. These files are included in the SAFE package.



Material Properties :

Modulus of Elasticity = 3000 ksi
Poisson's Ratio = 0.3

Loading :

Uniform Load, q = 100 psf

Figure 3-1
Rectangular Plate with Two Edges Simply Supported, One Edge Fixed and One Edge Free

Location		SAFE Displacement (in)			Theoretical Displacement (in)
X (in)	Y (in)	4x4 Mesh	8x8 Mesh	12x12 Mesh	
180	0	0.0000	0.0000	0.0000	0.00000
180	60	0.0849	0.0831	0.0827	0.08237
180	120	0.2410	0.2379	0.2372	0.23641
180	180	0.3971	0.3947	0.3940	0.39309
180	240	0.5537	0.5511	0.5502	0.54908

Table 3-1
Comparison of Displacements

Example 4

Rectangular Plate on Elastic Beams

Description

The plate, shown in Figure 4-1, is analyzed for a uniformly distributed surface load. The edges along $x = 0$ and $x = a$ are simply supported, and the other two edges are supported on elastic beams. It is assumed that the beams resist bending in vertical planes only and do not resist torsion. A theoretical solution to this problem is given in Timoshenko and Woinowsky (1959). The deflections of the plate and the moments and shears of the edge beams are compared with both the theoretical solution and the results obtained using the Direct Design Method as outlined in ACI 318-95 for $\lambda = 4$. λ is the ratio of the bending stiffness of the beam and the bending stiffness of the slab of width equal to the length of the beam and is given by the following equation.

$$\lambda = \frac{EI_b}{aD}, \text{ where,}$$

$$D = \frac{Eh^3}{12(1-\nu^2)},$$

I_b is the moment of inertia of the beam about the horizontal axis,

a is the length of the beam which is also equal to the one side of the slab, and

h is the thickness of the slab.

Data

Plate size,	$a \times b$	=	$360'' \times 240''$
Plate thickness,	T	=	8 inches
Modulus of elasticity,	E	=	3000 ksi
Poisson's ratio,	ν	=	0.3
Beam Moment of Inertia,	I_b	=	67520 in^4
Relative stiffness parameter,	λ	=	4
Load Case:	q	=	100 psf (Uniform load)

Modeling Procedure

To test convergence of results, the problem is analyzed employing three mesh sizes, 4×4, 8×8, and 12×12, as shown in Figure 1-2. The slab is modeled with thin plate elements. The simply supported edges are modeled as line supports with a large vertical stiffness and zero rotational stiffness. Beam elements, with no torsional rigidity, are defined on edges $y = 0$ and $y = b$. The flexural stiffness of edge beams is expressed as a factor λ of the plate flexural stiffness.

The subdivision of the plate into column and middle strips and also the definition of tributary loaded areas for shear calculations comply with ACI 318-95 provisions and shown in Figure 4-2. A load factor of unity is used and the self-weight of the plate is not included in the analysis.

Comparison of Results

As seen in Table 4-1, comparison of SAFE deflections for $\lambda = 4$ shows monotonic convergence to the theoretical values with successive mesh refinement.

The variation of bending moment in edge beams along its length is shown in Table 4-2 for $\lambda = 4$. The theoretical solution as well as the ACI approximation by the Direct Design Method is also shown. The quantity λ is analogous to the ACI ratio $\alpha_1 l_2 / l_1$ (refer to Sections 13.6.4.4 and 13.6.5.1 of ACI 318-95). The correlation between the numerical results from SAFE and the theoretical results is excellent. For design purposes, the ACI approximation (Direct Design Method) compares well with the theory. The moments were obtained at the grid points. In obtaining SAFE moments, averaging was done at the grid points.

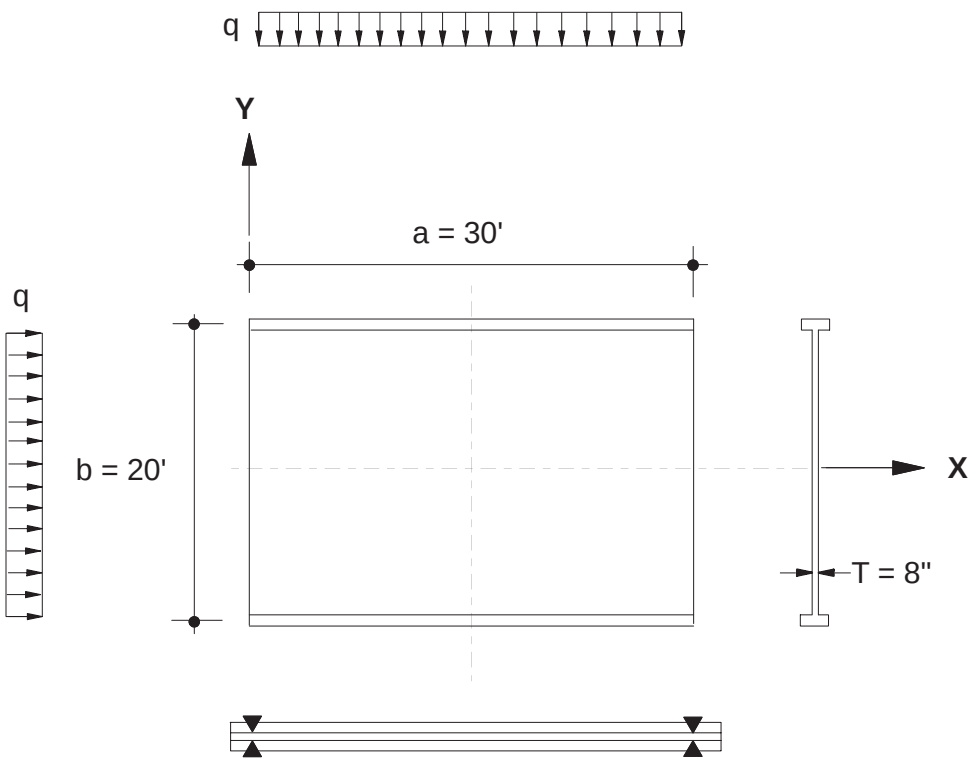
In obtaining the ACI moments, the following quantities are computed: $\alpha_1 = E_{cb} I_b / E_{cs} I_s = 6.59375$, $l_2 / l_1 = 240/360 = 0.667$, $\alpha_1 l_2 / l_1 = 4.3958$, $\beta_t = 0$, $M_0 = 2700$ Kip-in. From ACI section 13.6.4.4, for $l_2 / l_1 = 0.667$ and

$\alpha_1 l_2 / l_1 = 4.3958$, it is determined that the column strip supports 85% of the total positive moment. The beam and slab do not carry any negative moment in the long direction because of the simply supported boundary condition. From ACI section 13.6.5.1, for $\alpha_1 l_2 / l_1 = 4.3958$, it is determined that the beam carries 85% of the total column strip moment. Since one beam supports only one-half of the column strip, the maximum beam positive moment is 0.36125 (= $0.85 \times 0.85 \times 0.5$) times M_0 which is equal to 975.375 kip-in. The beam moments at other locations are obtained assuming a parabolic variation along the beam length.

Table 4-3 shows the variation of shear in edge beams for $\lambda = 4$. The agreement is excellent. The ACI values are calculated based on the definition of loaded tributary areas given in Section 13.6.8.1 of ACI 318-95. The shear forces were obtained at the middle of the grid points. In obtaining SAFE moments, no averaging was required for the shear forces.

File Reference

The files for the 4×4, 8×8, and 12×12 meshes are S04a.FDB, S04b.FDB, and S04c.FDB, respectively. These files are included in the SAFE package.



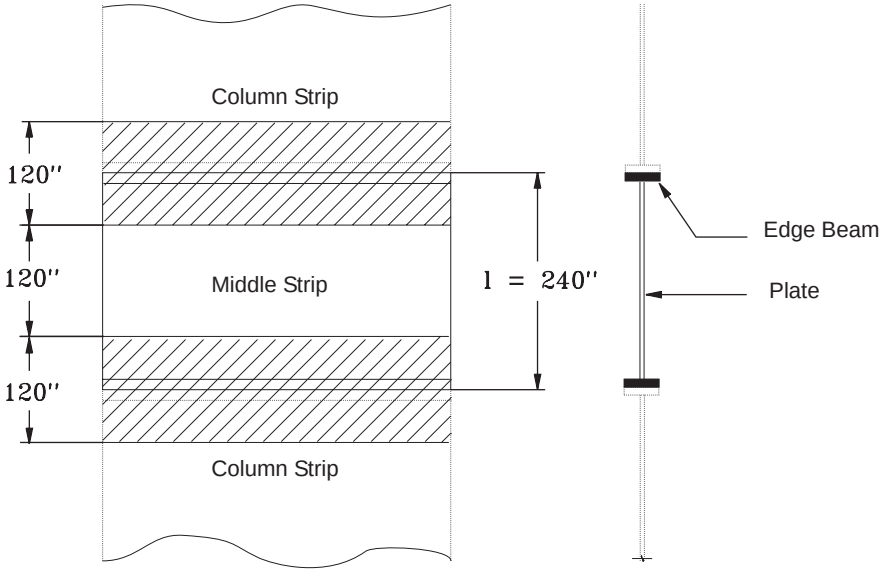
Material Properties :

Modulus of Elasticity = 3×10^6 psi
Poisson's Ratio = 0.3

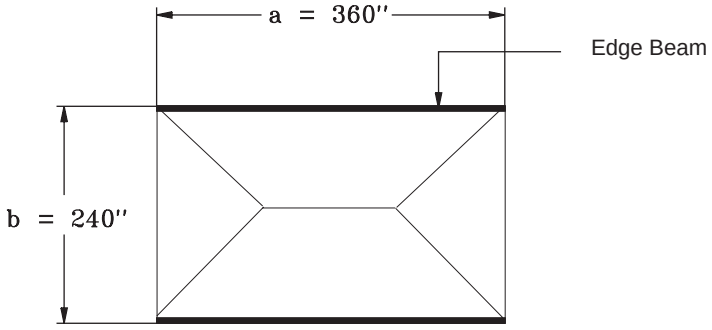
Loading :

Uniform Load, q = 100 psf

Figure 4-1
Rectangular Plate on Elastic Beams



Definition of Strips



Tributary Loaded Area for Shear on Edge Beams

Figure 4-2
Definition of Slab Strips and
Tributary Areas for Shear on Edge Beams

Location		SAFE Displacement (in)			Theoretical Displacement (in)
X (in)	Y (in)	4x4 Mesh	8x8 Mesh	12x12 Mesh	
180	0	0.1812	0.1848	0.1854	0.18572
180	60	0.1481	0.1523	0.1530	0.15349
180	120	0.0675	0.0722	0.0730	0.07365

Table 4-1
Comparison of Displacements

Location		Edge Beam Moment (Kip-in)				
Y (in)	**X (in)**	**4×4**	**8×8**	**12×12**	**ACI**	**Theoretical**
±120	0	0	0	0	0	0
	30	—	313.0	—	298.031	313.4984
	60	590.8	591.4	591.5	541.875	591.6774
	120	—	984.9	—	867.000	984.7026
	180	1120.9	1120.7	1120.4	975.375	1120.1518

Table 4-2
Variation of Bending Moment in an Edge Beam ($\lambda = 4$)

Location		Edge Beam Shear (Kip)				
Y (in)	**X (in)**	**4×4**	**8×8**	**12×12**	**ACI**	**Theoretical**
±120	10	—	—	10.58	9.9653	10.6122
	15	—	10.43	—	9.9219	10.4954
	30	9.80	—	9.96	9.6875	9.9837
	45	—	9.26	—	9.2969	9.2937
	50	—	—	9.02	9.1319	9.0336
	80	—	—	7.23	7.7778	7.2458
	90	4.40	6.55	—	7.1875	6.5854
	120	—	—	4.48	5.0000	4.4821
	150	—	2.26	—	2.5000	2.2656
	160	—	—	1.51	1.6667	1.5133

Table 4-3
Variation of Shear in an Edge Beam ($\lambda = 4$)

Example 5

Infinite Flat Plate on Equidistant Columns

Description

The plate, shown in Figure 5-1, is analyzed for uniform load. The overall dimensions of the plate are large in comparison with the column spacings *a* and *b*. Analysis is limited to a single interior panel because it can be assumed that deformation is identical in panels away from the boundaries. An analytical solution, based on the foregoing assumption, is given in Timoshenko and Woinowsky (1959).

The numerically computed deflection, local moments, and local shears obtained from SAFE are compared with their theoretical counterparts in Tables 5-1 through 5-3. The average design strip moments obtained from SAFE are compared with those obtained from two ACI alternative methods, the Direct Design Method and the Equivalent Frame Method, and the theoretical method. The comparison is shown in Table 5-4.

Data

Plate size,	$a \times b$	$= 360'' \times 240''$
Plate thickness,	T	= 8 inches
Modulus of elasticity,	E	= 3000 ksi
Poisson's ratio,	ν	= 0.3
Load Case: Uniform load,	q	= 100 psf

Modeling Procedure

Three mesh sizes, 4×4, 8×8, and 12×12, as shown in Figure 1-2, are used to test the convergence property of the model. The model consists of a plate of uniform thickness supported at four corners point. The effect of column support within a finite area is not modeled. Due to symmetry, the slope of the deflection surface in the direction normal to the boundaries are zero along the edges and the shearing force are zero at all points along the edges of the panel except at the corners. To model this boundary condition, line supports with a large rotational stiffness about the support line are defined on all four edges. Additional point supports are provided at the corners. The plate is modeled with the thin plate elements in SAFE. In doing so, the effect of shear distortion is neglected.

To approximately model the rigid corners, the slab thickness is increased for the 12×12 mesh to five times its original value in the region concerned, shown as the $40'' \times 40''$ areas in Figure 5-2.

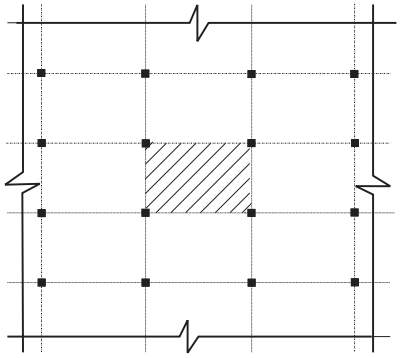
To obtain design moments, the panel is divided into three strips both ways, two column strips and one middle strip, based on the ACI 318-95 definition of design strip widths, as shown in Figure 5-2 and in Figure 5-3. A load factor of unity is used. The self-weight of the plate is not included in the analysis.

Comparison of Results

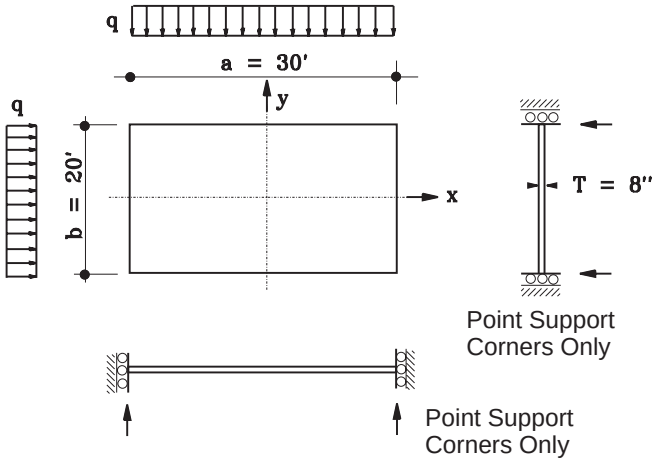
The numerical and the theoretical deflections are compared in Table 5-1. This table shows monotonic convergence of the numerical solution to the theoretical values with successive mesh refinement. The SAFE results for local moment and shear also compare closely with the theoretical values as shown in Table 5-2 and Table 5-3, respectively. EFM is used to calculate the interior span moments as depicted in Figure 5-2 and Figure 5-3. In Table 5-4, average strip moments obtained from SAFE are compared with both the ACI and the theoretical values. The effect of corner rigidity is shown in Table 5-5. The agreement between the SAFE and the theoretical solution is excellent. ACI approximations, employing either DDM or EFM, however, deviate from the theory. It should be noted that, regardless of the method used, the absolute sum of positive and negative moments in each direction adds up to the total static moment in that direction.

File Reference

The files for the 4×4, 8×8, and 12×12 meshes are S05a.FDB, S05b.FDB, and S05c.FDB, respectively. Also the file for studying corner rigidity is S05d.FDB. These files are included in the SAFE package.



Floor Plan



A Typical Bay

Modulus of Elasticity	= 3000	ksi
Poisson's Ratio	= 0.3	
Uniform Load	= 100	psf

Figure 5-1
Rectangular Plate on Equidistant Columns

Location		SAFE Displacement (in)			Theoretical Displacement
X (in)	Y (in)	4x4 Mesh	8x8 Mesh	12x12 Mesh	(in)
0	0	0.263	0.278	0.280	0.280
0	60	0.264	0.274	0.275	0.275
0	120	0.266	0.271	0.271	0.270
120	0	0.150	0.153	0.153	0.152
120	120	0.101	0.101	0.100	0.098
180	0	0.114	0.108	0.106	0.104
180	60	0.072	0.069	0.067	0.065
180	120	0.000	0.000	0.000	0.000

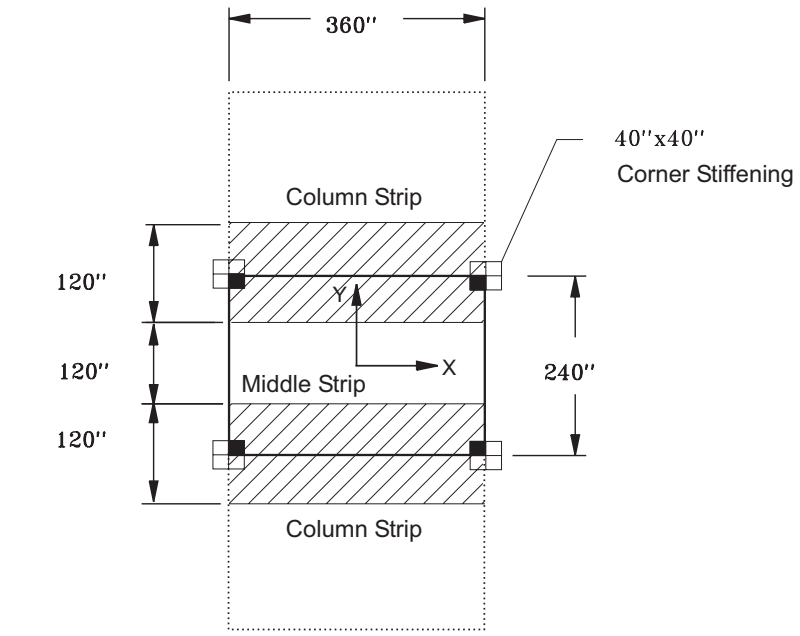
Table 5-1
Comparison of Displacements

Location		Moments (kip-in/in)			
		M_x		M_y	
X (in)	Y (in)	SAFE (8x8)	Theoretical	SAFE (8x8)	Theoretical
30	15	3.093	3.266	1.398	1.470
30	105	3.473	3.610	0.582	0.580
165	15	-2.948	-3.142	1.887	1.904
165	105	-9.758	-9.504	-7.961	-7.638

Table 5-2
Comparison of Local Moments

Location		Shear Force ($\times 10^{-3}$ kip/in)			
		V_x		V_y	
X (in)	Y (in)	SAFE (8x8)	Theoretical	SAFE (8x8)	Theoretical
30	15	20.0	17.3	3.5	2.2
30	105	21.2	23.5	3.1	5.4
165	15	17.3	14.7	19.1	23.8
165	105	357.1	329.0	350.4	320.0

Table 5-3
Comparison of Local Shears



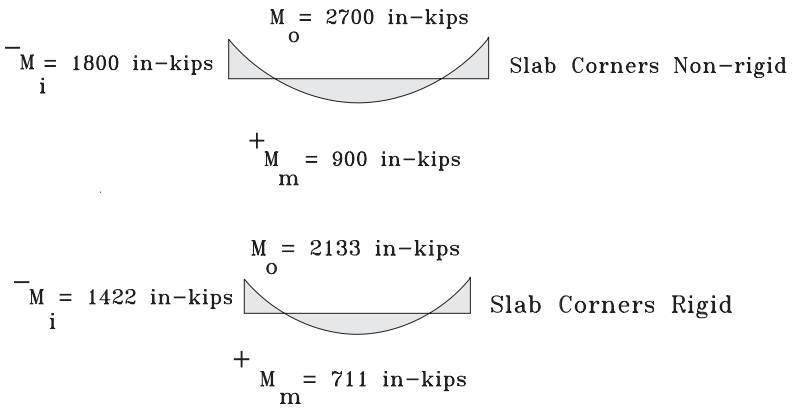


Figure 5-2
Definition of X-Strips (Moment values are obtained by EFM)

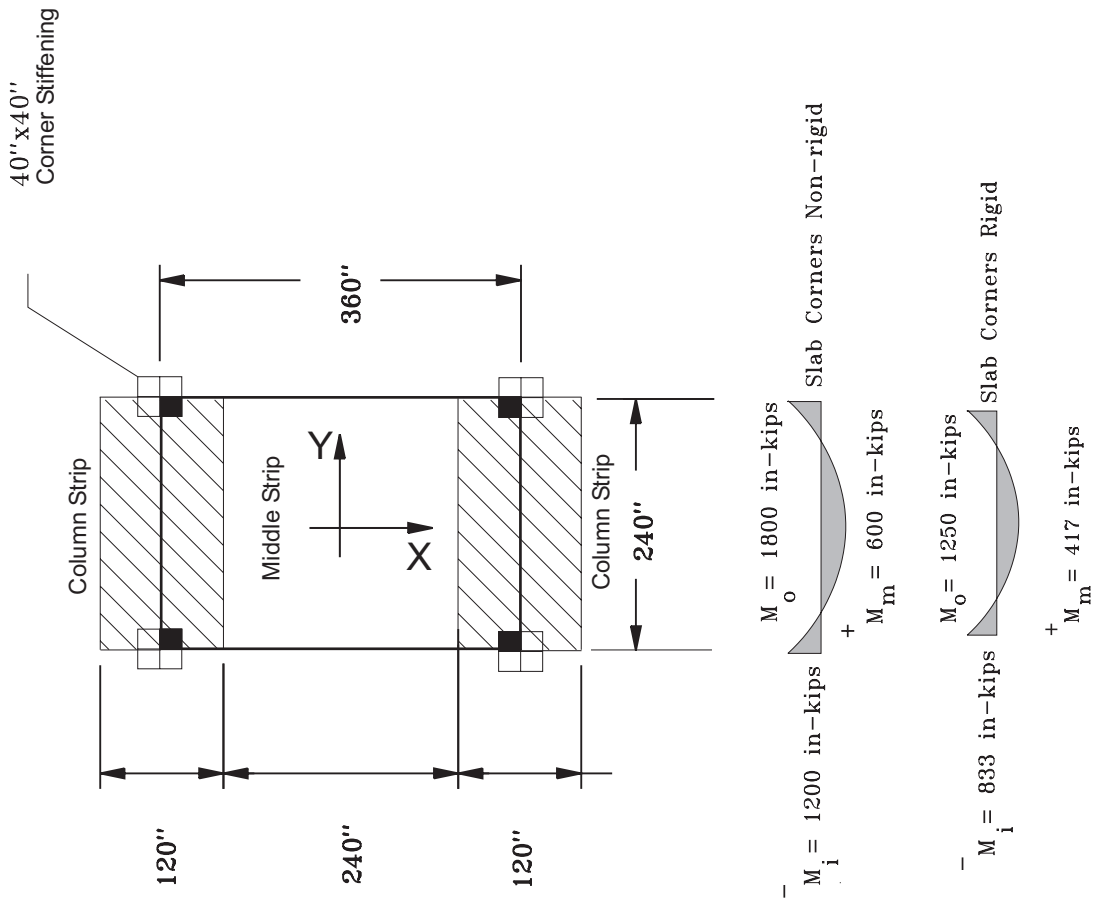


Figure 5-3
Definition of Y-Strips (Moment values are obtained by EFM)

Average Moment	Location	Strip	SAFE Moment (kip-in/in)			Theoretical (kip-in/in)	ACI 318-95 (kip-in/in)	
			4×4 Mesh	8×8 Mesh	12×12 Mesh		DDM	EFM
$\overline{M}_x$	$x = 0''$	Column	4.439	4.003	3.925	3.859	4.725	4.500
		Middle	4.311	3.809	3.714	3.641	3.150	3.000
$\overline{M}_x$	$x = 180''$	Column	-10.218	-10.906	-11.014	-11.109	-10.968	-11.250
		Middle	-3.532	-3.782	-3.847	-3.891	-3.656	-3.750
$\overline{M}_y$	$y = 0''$	Column	2.270	2.031	1.973	1.925	3.150	3.000
		Middle	1.677	1.563	1.548	1.538	1.050	1.000
$\overline{M}_y$	$y = 120''$	Column	-8.267	-8.941	-9.042	-9.139	-7.313	-7.500
		Middle	-0.554	-0.451	-0.444	-0.430	-1.219	-1.250

Table 5-4
Comparison of Average Strip Moments

Average Moment	Location	Strip	SAFE Moment (12×12 Mesh) (kip-in/in)		ACI 318-95 (EFM Method) (kip-in/in)	
			Slab Corner Non-rigid	Slab Corner Rigid	Slab Corner Non-rigid	Slab Corner Rigid
$\overline{M}_x$	$x = 0''$	Column	3.925	3.472	4.500	3.555
		Middle	3.714	3.286	3.000	2.370
$\overline{M}_x$	$x = 160''$	Column	-6.680	-8.195	—	-8.887
		Middle	-3.459	-2.824	—	-2.962
$\overline{M}_y$	$y = 0''$	Column	1.973	1.470	3.000	2.085
		Middle	1.548	1.362	1.000	0.695
$\overline{M}_y$	$y = 100''$	Column	-4.800	-5.553	—	-5.206
		Middle	-0.273	-0.335	—	-0.867

— *Not computed*

Table 5-5
Comparison of Average Strip Moments : Effect of Corner Rigidity

Example 6

Infinite Flat Plate on Elastic Subgrade

Description

An infinite plate resting on elastic subgrade and carrying equidistant and equal loads, P, is shown in Figure 6-1. Each load is assumed to be distributed uniformly over the area $u \times v$ of a rectangle. A theoretical double series solution to this example is given in Timoshenko and Woinowsky (1959).

The numerically computed deflections, local moments, and local shears obtained from SAFE are compared to the theoretical values as shown in Table 6-1 and Table 6-2.

Data

Plate size,	$a \times b$	$= 360'' \times 240''$
Plate thickness,	T	= 15 inches
Modulus of elasticity,	E	= 3000 ksi
Poisson's ratio,	ν	= 0.2
Modulus of subgrade reaction,	k	= 1 ksi/in

Loading: Point Load, P = 400 kips
(assumed to be uniformly distributed over an area $u \times v$)

Modeling Procedure

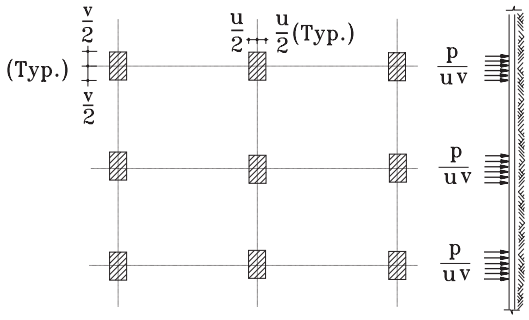
Analysis is confined to a single interior panel. Three mesh sizes, 4×4, 8×8, and 12×12 are used to model the panel as shown in Figure 1-2. The slab is modeled with thin plate elements and the elastic support is modeled as a surface support with a spring constant of k, the modulus of subgrade reaction. The edges are modeled as line supports with a large rotational stiffness about the support line. Point loads $P/4$ are defined at the panel corners. In the theoretical formulation (Timoshenko and Woinowsky 1959), each column load P is assumed to be distributed over an area $u\times v$ of a rectangle, as shown in Figure 6-1. To apply the theoretical formulation to this problem, concentrated corner loads are modeled as a uniformly distributed load acting over a very small rectangular area where u and v are very small.

Comparison of Results

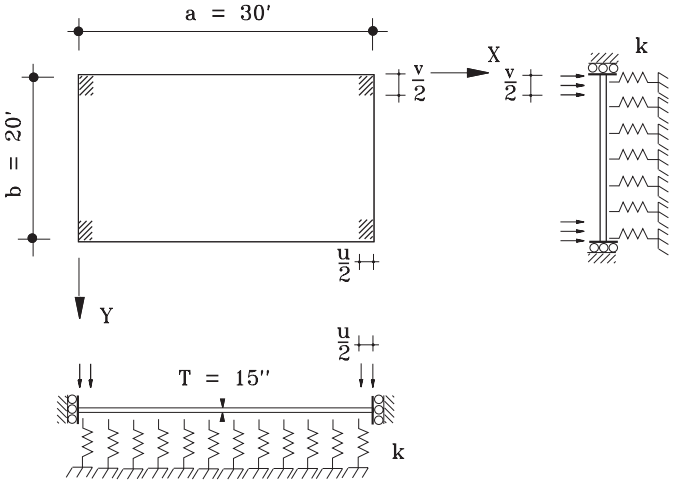
Excellent agreement has been found between the numerical and theoretical deflection for $k = 1$ ksi/in as shown in Table 6-1. As the modulus k is changed, the distribution of pressure between the plate and the subgrade changes accordingly. The particular case, as k approaches 0, corresponds to a uniformly distributed subgrade reaction, i.e., to the case of a "reversed flat slab" uniformly loaded with $q = P/ab$. In fact the problem changes to that of Example 5, with the direction of vertical axis reversed. In Example 5, for a uniform load of 100 psf ($P = 60$ kips), the maximum relative deflection is calculated as $0.280''$. Applying the formulation used here with $k = 1\times 10^{-6}$ yields a deflection value of $0.279''$. SAFE local moments using the 12×12 mesh have been compared with the theoretical results in Table 6-2. The results agree well.

File Reference

The files for the 4×4, 8×8, and 12×12 meshes are S06a.FDB, S06b.FDB, and S06c.FDB, respectively. These files are included in the SAFE package.



FLOOR PLAN



A TYPICAL BAY

Material Properties :

Modulus of Elasticity	= 3000	ksi
Poissons Ratio	= 0.2	
Subgrade Modulus	= 1	ksi/in

Loading :

Typical Column Load	= 400	kips

Figure 6-1
Rectangular Plate On Elastic Subgrade

Location		SAFE Displacement (in)			Theoretical Displacement (in)
X (in)	Y (in)	4x4 Mesh	8x8 Mesh	12x12 Mesh	
0	0	-0.04935	-0.05410	-0.05405	-0.05308
180	0	0.00180	0.00093	0.00095	0.00096
180	120	0.00040	0.00060	0.00064	0.00067

Table 6-1
Comparison of Displacements

Location		Moments (kip-in/in)			
		M_x		M_y	
X (in)	Y (in)	SAFE (12×12)	Theoretical	SAFE (12×12)	Theoretical
10	10	37.99	35.97	37.97	35.56
10	50	7.38	7.70	-6.74	-6.87
10	110	-0.30	-0.26	-5.48	-5.69
80	10	-6.52	-6.89	1.98	1.72
80	50	-3.58	-3.78	-0.93	-1.02
80	110	-0.88	-0.98	-1.86	-1.69

Table 6-2
Comparison of Local Moments

Example 7

Skew Plate with Mixed Boundary

Description

A skew plate under uniform load, as shown in Figure 7-1, is analyzed for two different support configurations. In the first case, all the edges are assumed to be simply supported. In the second case, the edges $y = 0$ and $y = b$ are released, i.e., the plate is assumed to be supported on its oblique edges only. A theoretical solution to this problem is given in Timoshenko and Woinowsky (1959). In both cases, the maximum deflection and the maximum moment are compared with the corresponding theoretical values.

Data

Plate size,	$a \times b$	=	$480'' \times 240''$
Plate thickness,	T	=	8 inches
Modulus of elasticity,	E	=	3000 ksi
Poisson's ratio,	ν	=	0.2
Load Cases:Uniform load,	q	=	100 psf

Modeling Procedure

A 8×24 base mesh is used to model the plate as shown in Figure 7-1. A large vertical stiffness is defined for supports, and support-lines are added on all four edges

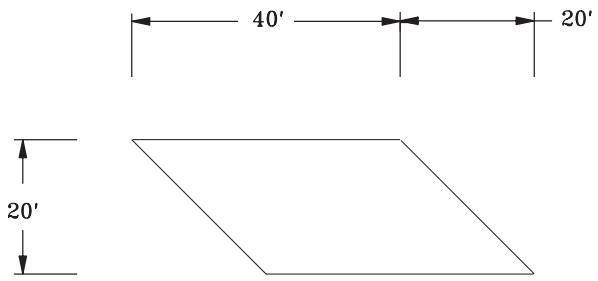
for the first case and along the skewed edges only for the second case. A load factor of unity is used. The self-weight of the plate is not included in the analysis.

Comparison of Results

The comparison of SAFE and the theoretical results is excellent, as shown in Table 7-1. Under the simply supported boundary condition, maximum deflection occurs at the plate center and the maximum principal moment acts nearly in the direction of the short span. Under the simply supported condition on the oblique edges and free boundary conditions on the other two edges, maximum deflection occurs at the free edge as expected.

File Reference

The files for two different boundary conditions are S07a.FDB and S07b.FDB. These files are included in the SAFE package.



Geometry

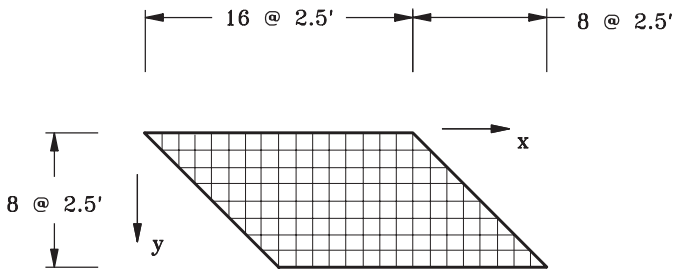
Material Properties :

Modulus of Elasticity = 3000 ksi

Poisson's Ratio = .2

Loading :

Uniform Load, q = 100 psf



Mesh

Support Conditions :

(1) Simply Supported on All Edges

(2) Simply Supported on Oblique Edges

Figure 7-1
Skew Plate

Boundary Condition	Responses	SAFE	Theoretical
Simply supported on all edges	Maximum displacement (inches)	0.156	0.162
	Maximum bending moment (kip-in)	3.66	3.59
Simply supported on oblique edges	Maximum displacement at the free edges (inches)	1.51	1.50
	Maximum bending moment at the free edges (kip-in)	12.16	11.84
Simply supported on oblique edges	Displacement at the center (inches)	1.21	1.22
	Maximum bending moment at the center (kip-in)	11.78	11.64

Table 7-1
Comparison of Deflections and Bending Moments

Example 8

ACI Handbook Flat Slab Example 1

Description

The flat slab system arranged three-by-four is shown in Figure 8-1. The slab consists of twelve 7.5 inch thick $18' \times 22'$ panels. Edge beams on two sides extend 16 inches below the slab soffit. Details are shown in Figure 8-2. There are three sizes of columns and in some locations, column capitals. Floor to floor heights below and above the slab are 16 feet and 14 feet respectively. A full description of this problem is given in Example 1 of ACI 340.R-97 (ACI Committee 340, 1997). The total factored moments in an interior E-W design frame obtained from SAFE are compared with the corresponding results obtained by the Direct Design Method, the Modified Stiffness Method, and the Equivalent Frame Method.

Data

Materials:

Concrete strength,	f_c'	=	3	ksi
Yield strength of steel,	f_y	=	40	ksi
Concrete unit weight,	γ_c	=	150	pcf
Modulus of elasticity,	E_c	=	3320	ksi
Poisson's ratio,	ν	=	0.2	

Loading:

Live load,	w_l	= 125	psf
Mechanical load,	w_{mech}	= 15	psf
Exterior wall load,	w_{wall}	= 400	plf

Modeling Procedure

The computational model uses a 10×10 mesh of elements per panel, as shown in Figure 8-3. The mesh contains gridlines at column centerlines, column faces, and the edges of column capitals. The grid-lines extend to the slab edges. The regular slab thickness is $7.5^{''}$. A slab thickness of $21.5^{''}$ is used to approximately model a typical capital. The slab is modeled with thin plate elements. The columns are modeled as point supports with vertical and rotational stiffnesses. Stiffness coefficients used in the calculation of support flexural stiffness are all reproduced from ACI Committee 340 (1997). Beams are defined on two slab edges as shown in Figure 8-1. The value of torsional constant (10767 in^4) used for the beams is also reproduced from ACI Committee 340 (1997).

The model is analyzed for uniform factored load of 0.365 ksf ($w_u = 1.4w_d + 1.7w_l$) in total including self-weight. To obtain factored moments in an E-W interior design frame, the slab is divided into strips in the X-direction (E-W direction) as shown in Figure 8-4. An interior design frame consists of one column strip and two halves of adjacent middle strips.

Comparison of Results

The SAFE results for the total factored moments in an interior E-W design frame are compared in Figure 8-5 with the results obtained by the Direct Design Method (DDM), the Modified Stiffness Method (MSM), and the Equivalent Frame Method (EFM). Only uniform loading with load factors of 1.4 and 1.7 has been considered. The DDM, MSM, and EFM results are all reproduced from Example 1 of ACI Committee 340 (1997), the Alternative Example 1 of ACI Committee 340 (1991), and from Example 3 of ACI Committee 340 (1991), respectively. Moments reported are calculated at the face of column capitals. Overall, they compare well. A noticeable discrepancy is observed in the negative column moment in the west side of the exterior bay (the edge beam side). In contrast to the EFM, the DDM appears to underestimate this moment. The SAFE result falls in between the two extreme values. The basic cause of this discrepancy is the way in which each method accounts for the combined flexural stiffness of columns framing into the joint. The DDM uses a stiffness coefficient k_c of 4 in the calculation of column and slab flex-

ural stiffnesses. The EFM, on the other hand, uses higher value of k_c to allow for the added stiffness of the capital and the slab-column joint. The use of MSM affects mainly the exterior bay moments which is not the case when the DDM is employed. In SAFE, member contributions to joint stiffness are dealt with more systematically than any of the above mentioned approaches. Hence, the possibility of overdesigning or underdesigning a section is greatly reduced.

The factored strip moments are compared in Table 8-1. There is a discrepancy in the end bays, particularly on the edge beam (west) side, where the SAFE and EFM results for exterior negative column strip moment show the greatest difference. This is expected because EFM simplifies a 3D structure to a 2D structure, thereby neglecting the transverse interaction between adjacent strips. Except for this localized difference the comparison is good.

File Reference

The file for this example is S08.FDB which is included in the SAFE package.

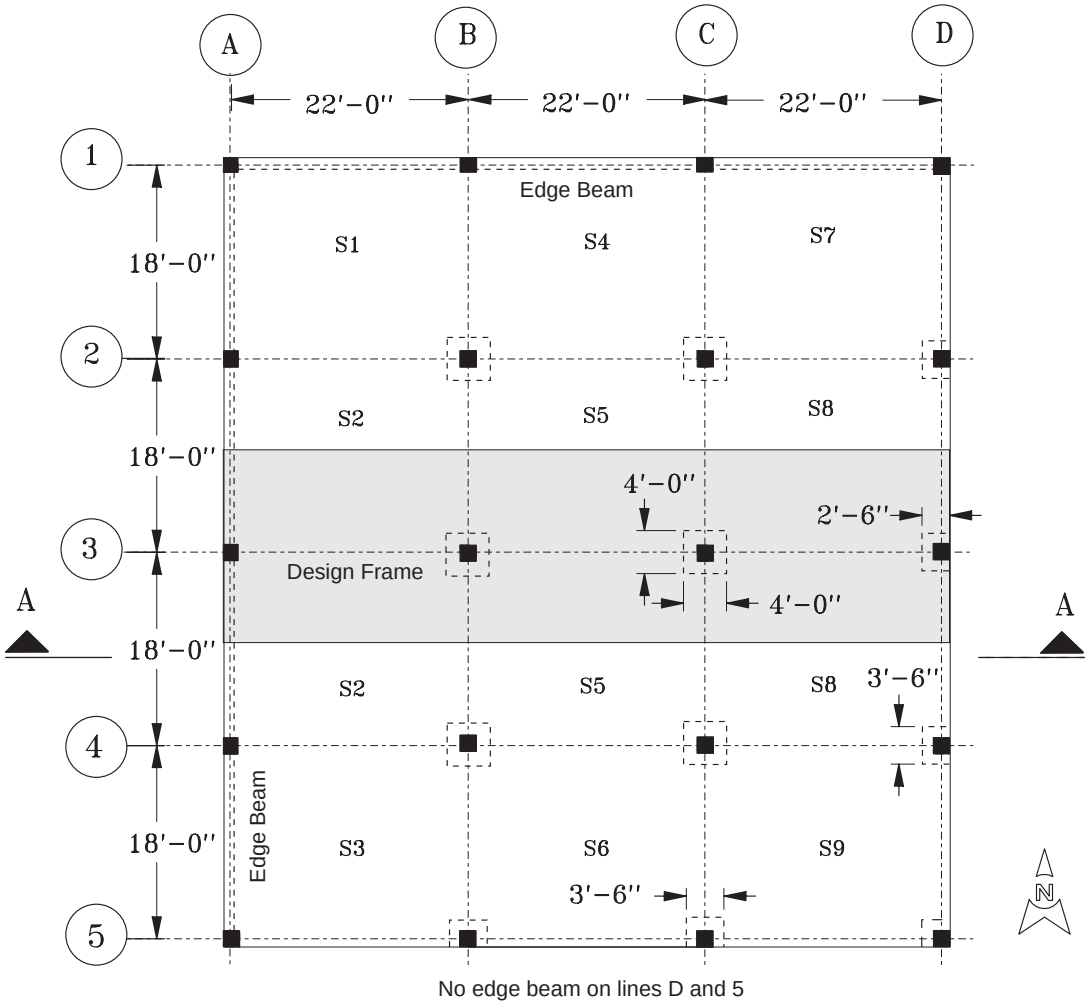


Figure 8-1
Flat Slab From ACI Handbook

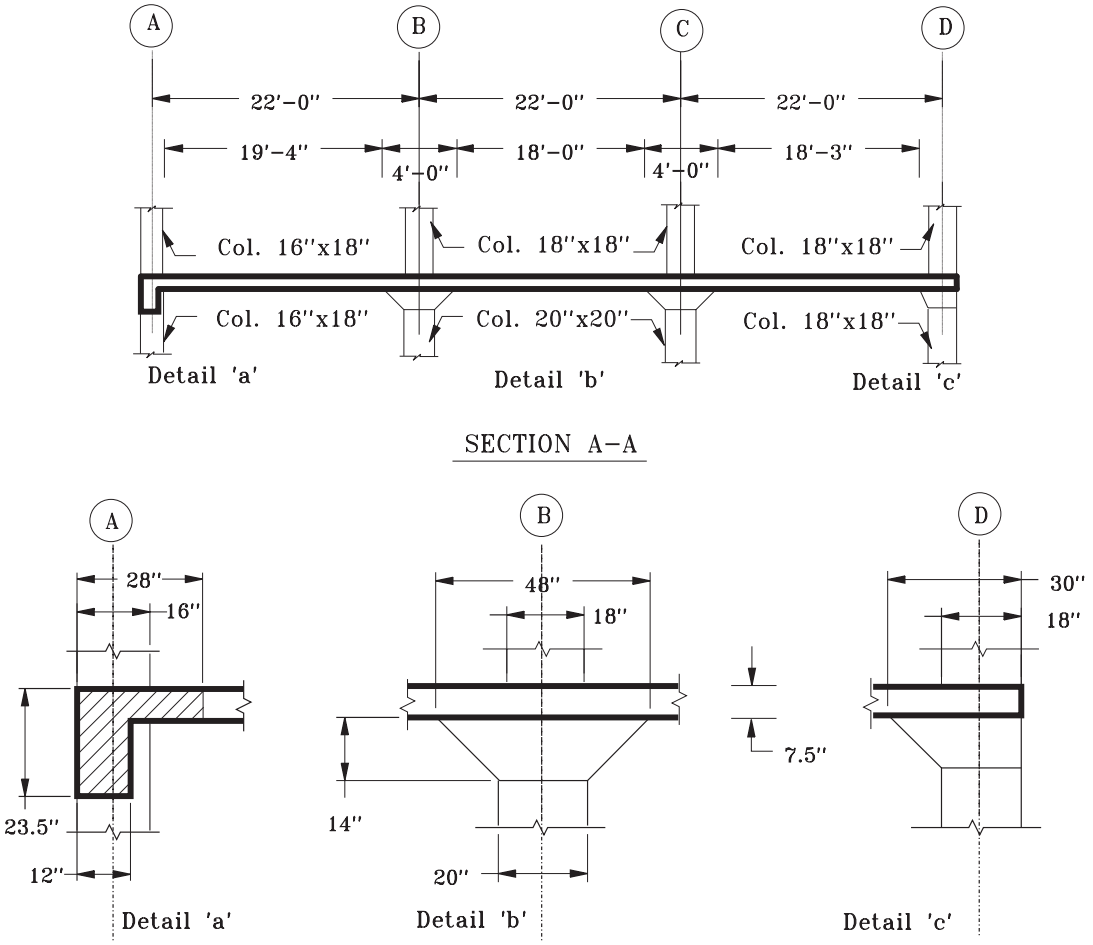


Figure 8-2
Sections and details of ACI Handbook Flat Slab Example

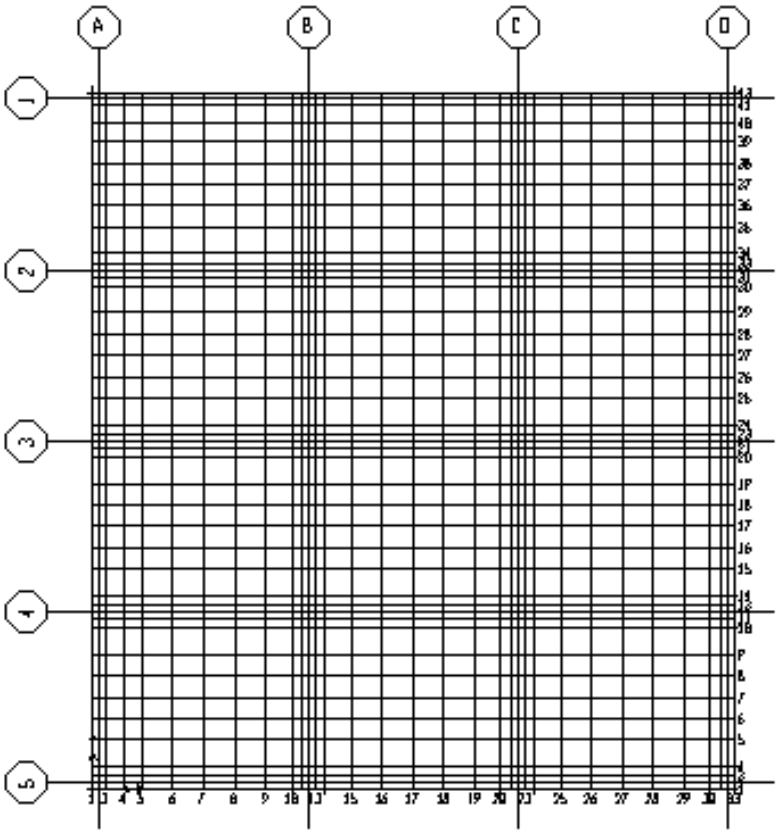


Figure 8-3
SAFE Mesh (10×10 *per panel)*

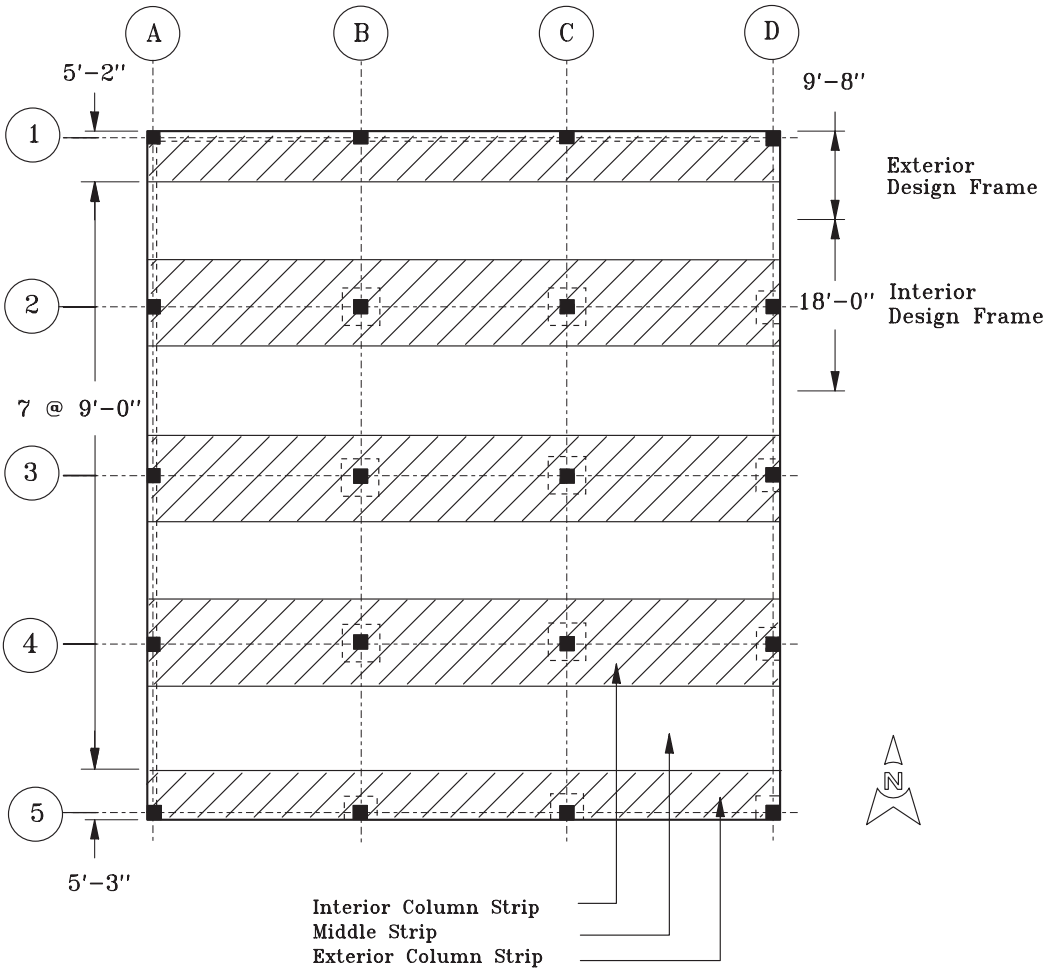


Figure 8-4
Definition of E-W Design Frames and Strips

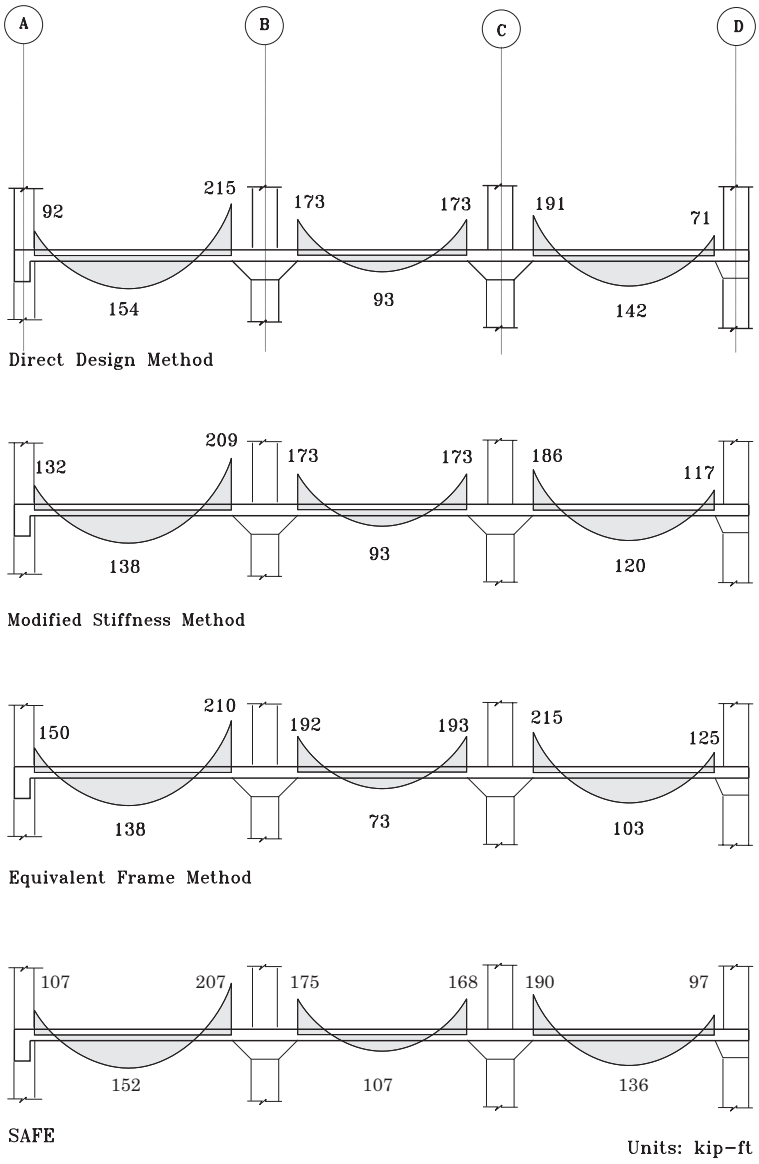


Figure 8-5
Comparison of Total Factored Moments (E-W Design Frame)

Strip	Method	Factored Strip Moment (kip-ft)								
		Span AB			Span BC			Span CD		
		^{-}M	^{+}M	^{-}M	^{-}M	^{+}M	^{-}M	^{-}M	^{+}M	^{-}M
Column Strip	DDM	86	92	161	130	56	130	143	85	71
	MSM	122	83	157	130	56	130	140	72	117
	EFM	140	83	157	144	44	145	161	62	125
	SAFE	78	79	161	132	57	128	148	72	95
Middle Strip	DDM	6	62	54	43	37	43	48	57	0
	MSM	10	55	52	43	37	43	46	48	0
	EFM	10	55	53	48	29	48	54	41	0
	SAFE	29	73	46	43	50	39	42	64	2

Table 8-1
Comparison of Total Factored Strip Moments (kip-ft)
(Interior E-W Design Frame)

Example 9

ACI Handbook Two-way Slab Example 2

Description

The two-way slab system arranged three-by-three is shown in Figure 9-1. The slab consists of nine 6.5 inch thick $20' \times 24'$ panels. Beams extend 12 inches below the slab soffit. Details are shown in Figure 9-2. $16'' \times 16''$ columns are used throughout the system. Floor to floor height is 15 ft. A full description of this problem is given in Example 2 of ACI 340.R-91 (ACI Committee 340, 1991). The total factored moments in an interior design frame obtained from SAFE are compared with the Direct Design Method, the Modified Stiffness Method, and the Equivalent Frame Method.

Data

Concrete strength,	f_c'	= 3	ksi
Yield strength of steel,	f_y	= 40	ksi
Concrete unit weight,	w_c	= 150	psf
Modulus of elasticity,	E_c	= 3120	ksi
Poisson's ratio,	ν	= 0.2	
Live load,	w_l	= 125	psf
Mechanical load,	w_d	= 15	psf
Exterior wall load,	w_{wall}	= 400	plf

Modeling Procedure

The computational model uses a 10×10 mesh of elements per panel, as shown in Figure 9-3. The mesh contains grid lines at both column centerlines and column faces. The grid lines are extended to the slab edges. The slab is modeled with thin plate elements. The columns are modeled as point supports with vertical and rotational stiffnesses. A stiffness coefficient of $4EI/L$ is used in the calculation of support flexural stiffness. Torsional constants of 4790 in^4 and 5478 in^4 are defined for the edge and interior beams respectively, in accordance with Section 13.7.5 of ACI 318-89 and Section 13.0 of ACI 318-95 code. The model is analyzed for uniform factored total load of 0.347 ksf $(w_u = 1.4w_d + 1.7w_l)$ including self-weight. To obtain factored moments in an interior design frame, the slab is divided into strips in X-direction (E-W direction) as shown in Figure 9-4. An interior design frame consists of one column strip and two halves of adjacent middle strips.

Comparison of Results

The SAFE results for the total factored moments in an interior E-W design frame are compared with the results obtained by the Direct Design Method (DDM), the Modified Stiffness Method (MSM), and the Equivalent Frame Method (EFM) as shown in Figure 9-5. The results are for uniform loading with load factors. The results are reproduced from ACI Committee 340 (1991). Moments reported are calculated at the column face. For all practical purposes they compare well. At the end bays, the MSM appears to overestimate the exterior column negative moments with the consequent reduction in the mid-span moments.

The distribution of total factored moments to the beam, column strip, and middle strip is shown in Table 9-1. The middle strip moments compare well. The total column strip moments also compare well. The distribution of the column strip moments between the slab and the beam has a larger scatter.

File Reference

The file for this example is S09.FDB which is included in the SAFE package.

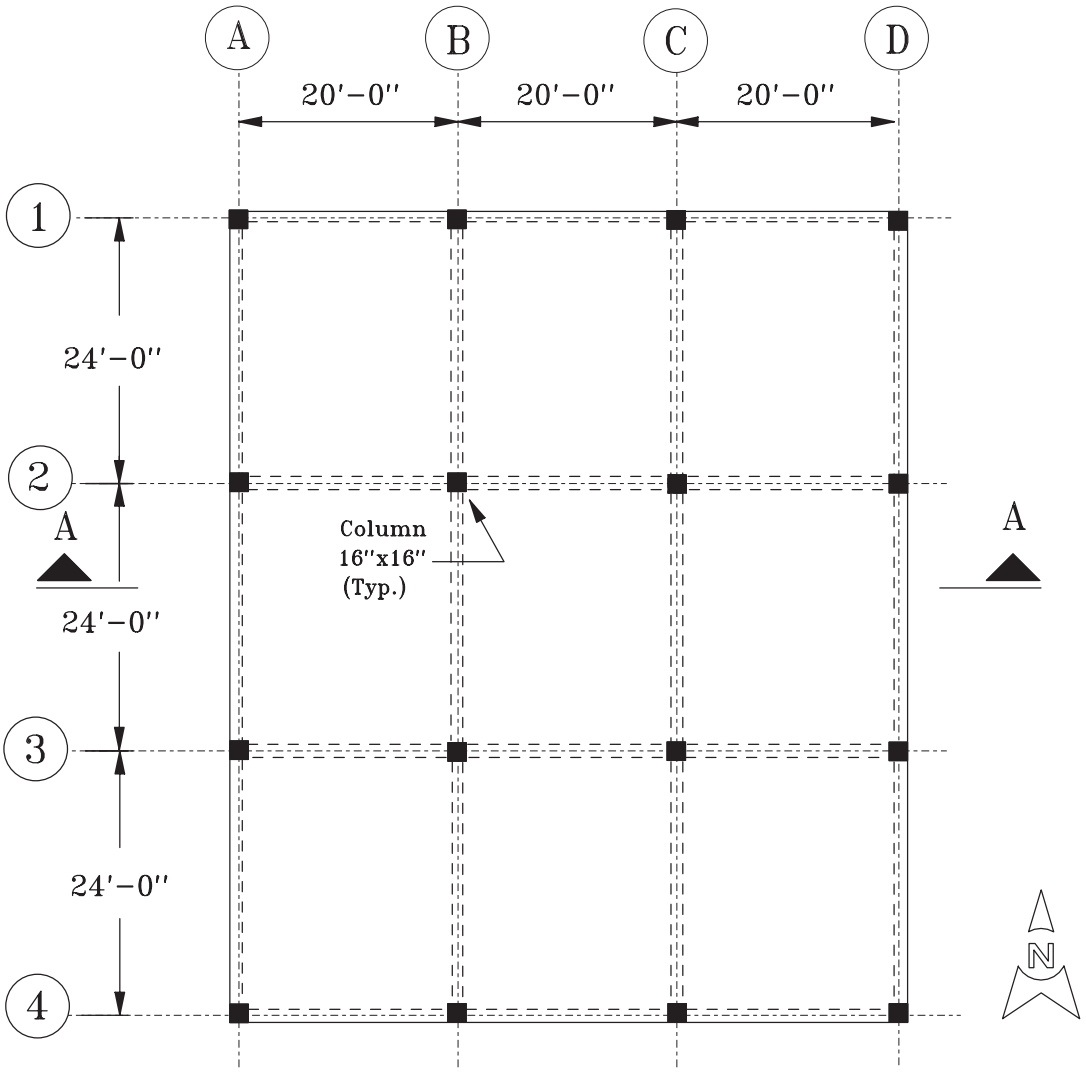


Figure 9-1
ACI Handbook Two-way Slab Example

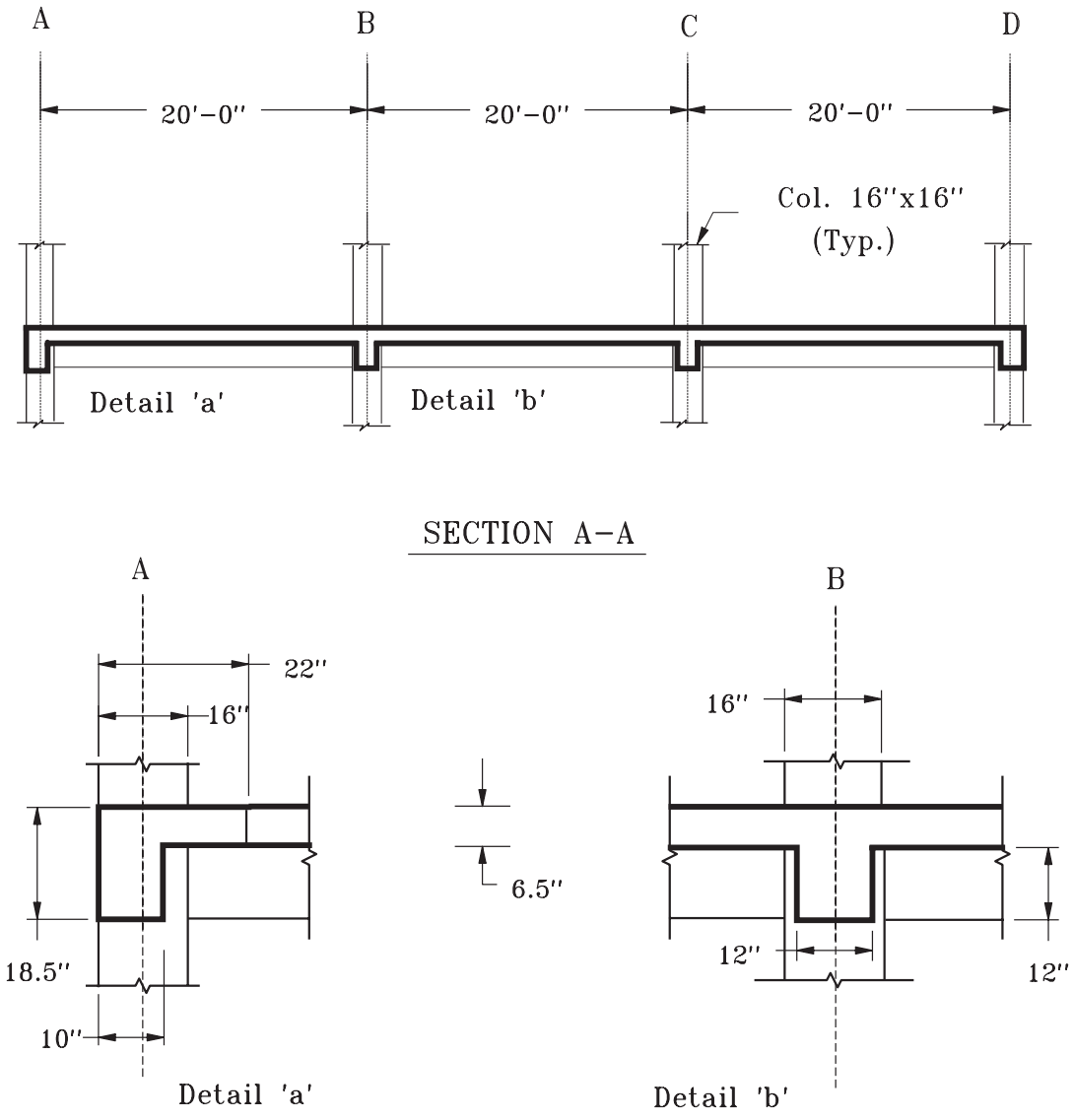


Figure 9-2
Details of Two-way Slab Example from ACI Handbook

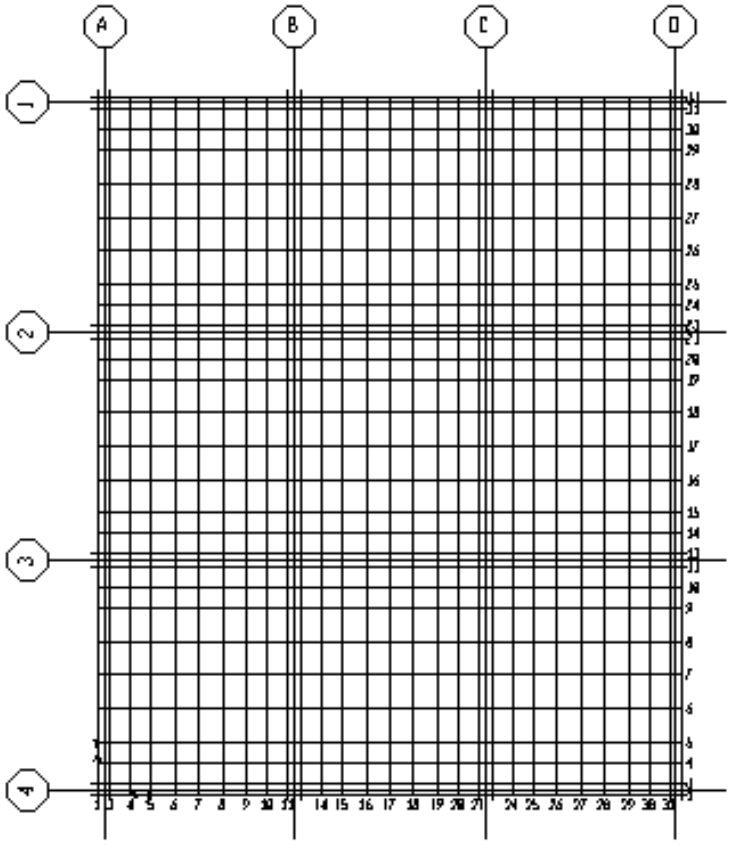


Figure 9-3
*SAFE Mesh (*10×10 *per panel)*

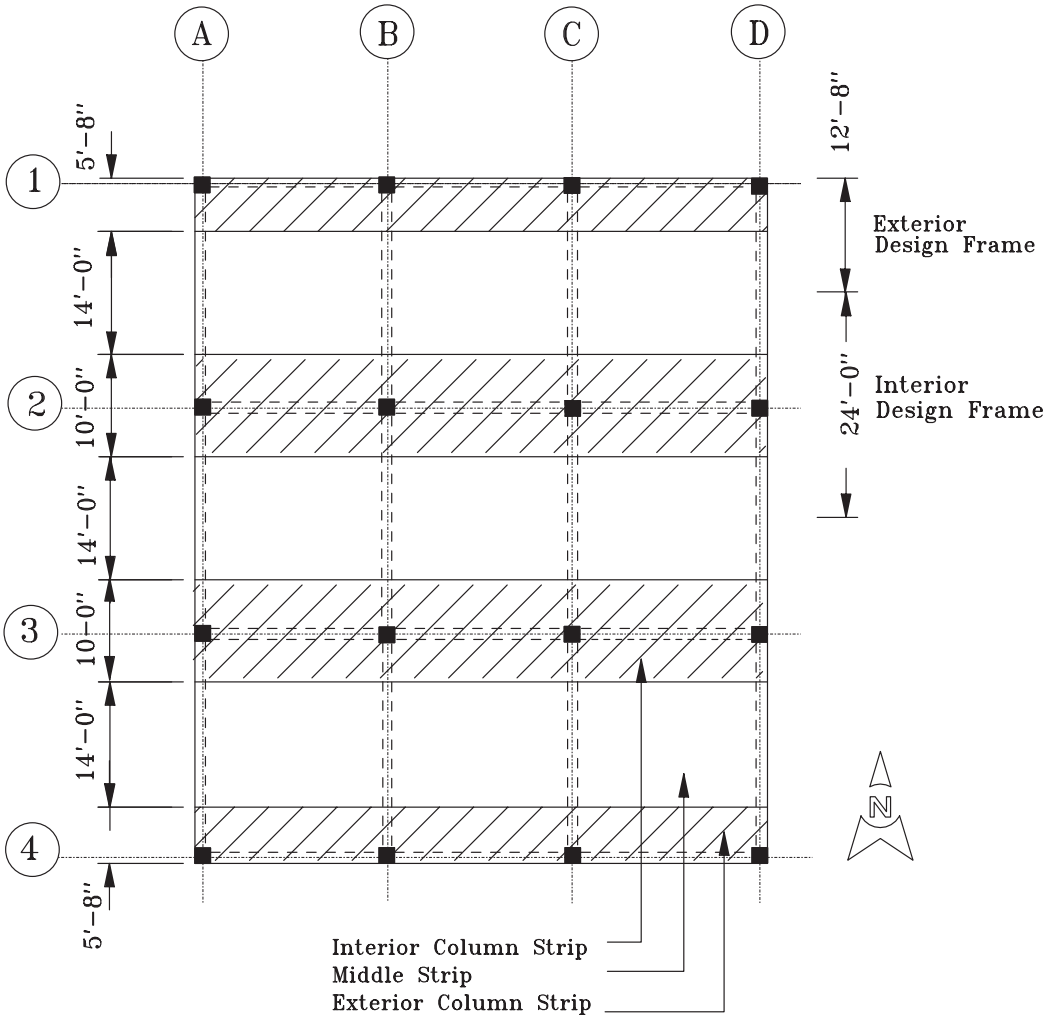


Figure 9-4
Definition of E-W Design Frames and Strips

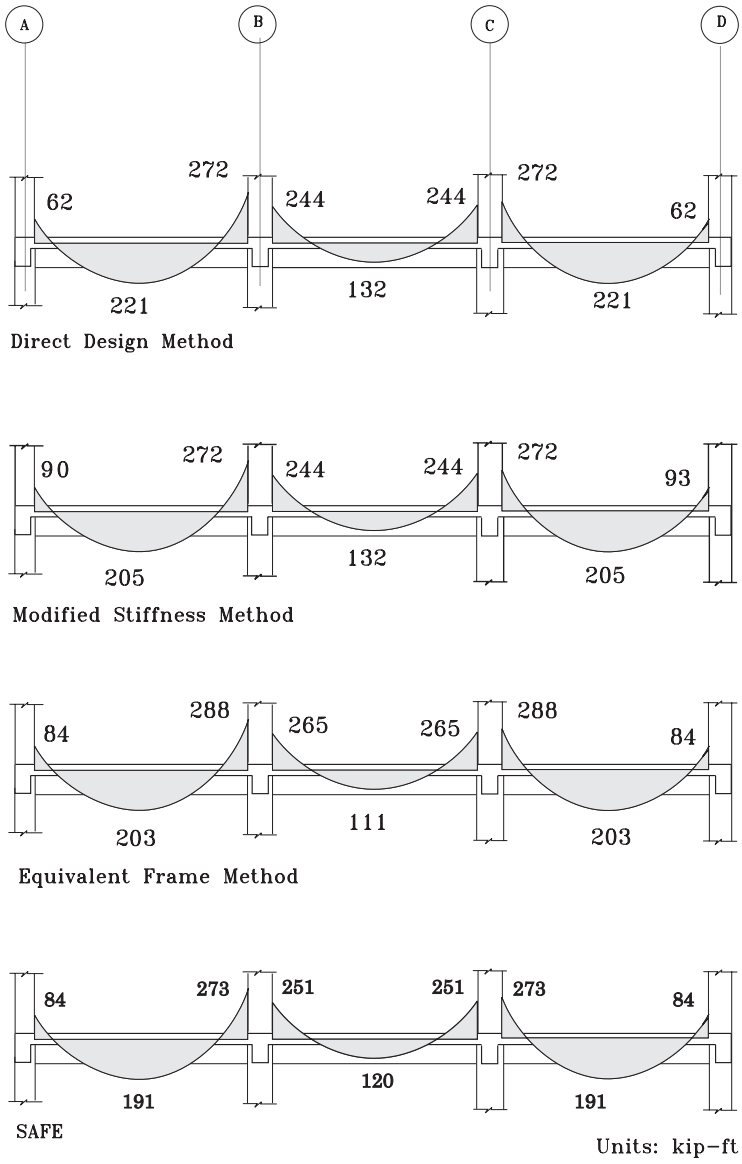


Figure 9-5
Comparison of Total Factored Moments (kip-ft) in an Interior E-W Design Frame

Beam / Slab Strip	Method	Total Factored Moments in an E-W Design Frame (kip-ft)					
		Exterior Span			Interior Span		
		^{-}M	^{+}M	^{-}M	^{-}M	^{+}M	^{-}M
Slab Column Strip	DDM	9	23	28	25	14	25
	MSM	13	21	28	25	14	25
	EFM	12	21	30	27	11	27
	SAFE	24	27	62	58	14	58
Slab Middle Strip	DDM	3	69	84	76	41	76
	MSM	5	63	84	76	41	76
	EFM	4	63	89	82	34	82
	SAFE	8	65	72	71	46	71
Beam	DDM	50	129	160	143	77	143
	MSM	72	121	160	143	77	143
	EFM	68	119	169	156	66	156
	SAFE	53	100	140	123	60	123

Table 9-1
Comparison of Total Factored Moments (kip-ft)

Example 10

PCA Flat Plate Test

Description

This example models the flat plate structure tested by the Portland Cement Association (Guralnick and LaFraugh, 1963). The structure consists of nine 5.25″ thick $15' \times 15'$ panels arranged 3×3 as shown in Figure 10-1. Deep and shallow beams are used on the exterior edges. The structure is symmetric about the diagonal line through columns A1, B2, C3, and D4, except the columns themselves are not symmetric about this line. The corner columns are $12'' \times 12''$ and the interior columns are $18'' \times 18''$. Columns along the edges are $12'' \times 18''$ with the longer dimension parallel to the plate edge. A typical section of the plate and details of edge beams are given in Figure 10-2. The total moments in an interior frame obtained numerically from SAFE are compared with the test results and the numerical values obtained by the Equivalent Frame Method (EFM).

Data

Concrete strength,	f_c'	= 4.1	ksi
Yield strength of steel,	f_y	= 40	ksi
Concrete unit weight,	w_c	= 150	pcf
Modulus of elasticity,	E_c	= 3670	ksi
Poisson's ratio,	ν	= 0.2	
Live load,	w_l	= 70	psf
Dead load,	w_d	= 86	psf

Modeling Procedure

A finite element model, shown in Figure 10-3, with 6×6 mesh per panel is employed in the analysis. The slab is modeled using the thin plate elements in SAFE. The columns are modeled as point supports with vertical and rotational stiffnesses. The reduced-height columns in the test structure are fixed at the base. Hence, rotational stiffnesses of point supports are calculated using a stiffness coefficient of 4 and an effective height of 39.75 inches ($K_c = 4EI / l_c$). The calculation of torsional stiffness of edge beams is based on the requirements of Section 13.7.5 of ACI 318-95. The model extends to the centerlines of the edge columns only. The portion of slab occupying the column area is modeled as rigid by increasing its thickness to five times the nominal slab thickness. A total uniformly distributed design load of 156 psf (not factored) is applied to all the panels.

To obtain design moment coefficients, the plate is divided into column and middle strips. An interior design frame consists of one column strip and half of each adjacent middle strip. Normalized values of design moments are used in the comparison.

Comparison of Results

The SAFE results for the total non-factored moments in an interior frame are compared with test results and the Equivalent Frame Method (EFM). The test and EFM results are all obtained from Corley and Jirsa (1970). The moments are compared in Table 10-1. The negative design moments reported are at the faces of the columns. Overall, the agreement between the SAFE and EFM results is good. The experimental negative moments at exterior sections, however, are comparatively lower. This may be partially the result of a general reduction of stiffness due to cracking in the beam and column connection at the exterior column which is not accounted for in an elastic analysis. It is interesting to note that even with an approximate representation of the column flexural stiffness, the comparison of negative exterior moments between EFM and SAFE is excellent.

File Reference

The file for this example is S10.FDB which is included in the SAFE package.

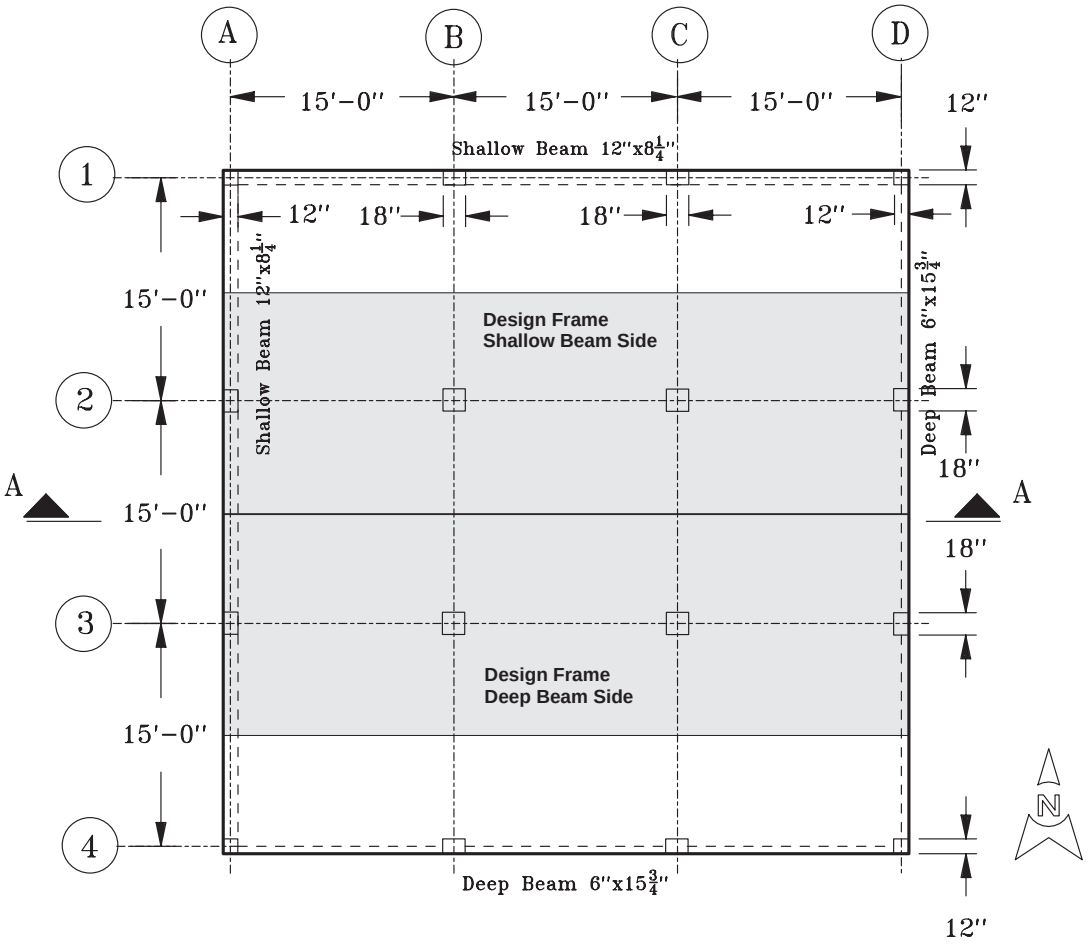


Figure 10-1
PCA Flat Plate Example

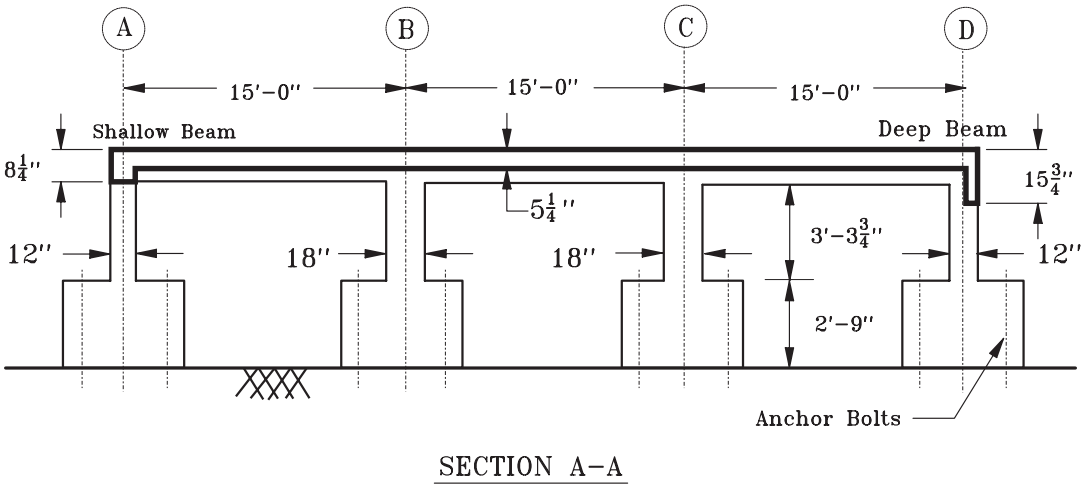


Figure 10-2
Section and Details of PCA Flat Plate Example

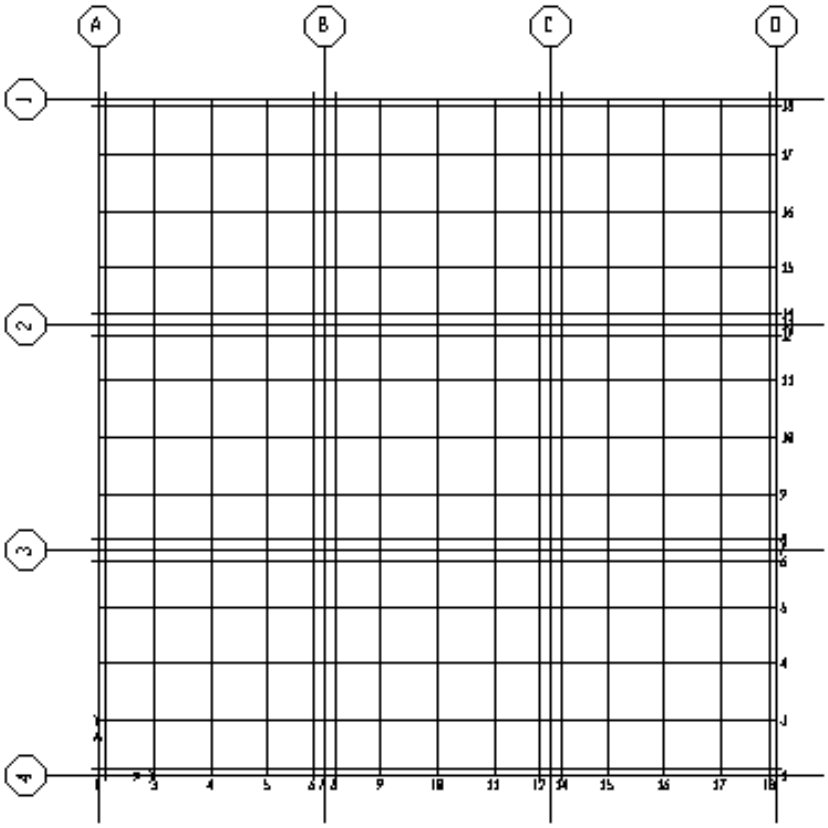


Figure 10-3
SAFE Mesh (6 × 6 per panel)

Method	Moments in an Interior Design Frame (M/Wl_1 *)								
	End Span (Shallow Beam Side)			Middle Span			End Span (Deep Beam Side)		
	^{-}M	^{+}M	^{-}M	^{-}M	^{+}M	^{-}M	^{-}M	^{+}M	^{-}M
PCA Test	0.037	0.047	0.068	0.068	0.031	0.073	0.073	0.042	0.031
EFM	0.044	0.048	0.067	0.062	0.038	0.062	0.068	0.049	0.043
SAFE (Shallow Beam Side)	0.044	0.050	0.068	0.062	0.041	0.062	0.067	0.051	0.042
SAFE (Deep Beam Side)	0.043	0.050	0.067	0.062	0.041	0.061	0.067	0.050	0.042

* $Wl_1 = 526.5$ *kip-ft*

Table 10-1
Comparison of Measured and Computed Moments

Example 11

University of Illinois Flat Plate Test F1

Description

This example models the flat plate structure tested by the University of Illinois by Hatcher, Sozen, and Siess (1965). The structure consists of nine $1.75^{\prime\prime}$ thick $5^{\prime} \times 5^{\prime}$ panels arranged three-by-three as shown in Figure 11-1. Two adjacent edges are supported by $2.00^{\prime\prime} \times 5.25^{\prime\prime}$ deep beams and the other two edges by shallow beams, 4 in. wide by 2.75 in. deep, producing a single diagonal line of symmetry through columns A1, B2, C3, and D4. A typical section and details of columns and edge beams are shown in Figure 11-2. The moments computed numerically using SAFE are compared with the test results and the EFM results.

Data

Material:

Concrete strength,	$f_c^{\prime}$	= 2.5	ksi
Yield strength of steel,	f_y	= 36.7	ksi
Modulus of elasticity,	E_c	= 2400	ksi
Poisson's ratio,	ν	= 0.2	

Loading:

Total uniform load	w	= 140	psf

Modeling Procedure

The computational model uses a 6×6 mesh of elements per panel as shown in Figure 11-3. The mesh contains gridlines at column centerlines as well as column faces. The slab is modeled with thin plate elements and the columns are modeled as point supports with vertical and rotational stiffnesses. The reduced-height columns in the test structure are pinned at the base. Hence, an approximate value of 3 $(K_c = 3EI/l_c)$ is used to calculate flexural stiffness of the supports taking the column height as $9.5''$. The slab is thickened over the column sections to account for rigidity of the slab-column joint. Shallow and deep beams are defined on the edges with torsional constants of 16 in^4 and 14.9 in^4, respectively, as described in Section 13.0 of ACI 318-95. The model is analyzed for uniform total load of 140 psf.

To obtain maximum factored moments in an interior design frame, the plate is divided into columns and middle strips. An interior design frame consists of one column strip and half of each adjacent middle strip.

Comparison of Results

The SAFE results for uniform load moments for an interior frame are compared with the experimental and EFM results in Table 11-1. The experimental and EFM results are all obtained from Corley and Jirsa (1970). The negative design moments reported are at the faces of the columns. From a practical standpoint, even with a coarse mesh the agreement between the SAFE and EFM results is good. In general the experimentally obtained moments at exterior sections are low, implying a loss of stiffness in the beam-column joint area.

In comparing absolute moments at a section, the sum of positive and average negative moments in the bay should add up to the total static moment. The SAFE and EFM results comply with this requirement within an acceptable margin of accuracy. The experimental results are expected to show greater discrepancy because of the difficulty in taking accurate strain measurements.

File Reference

The file for this example is S11.FDB which is included in the SAFE package.

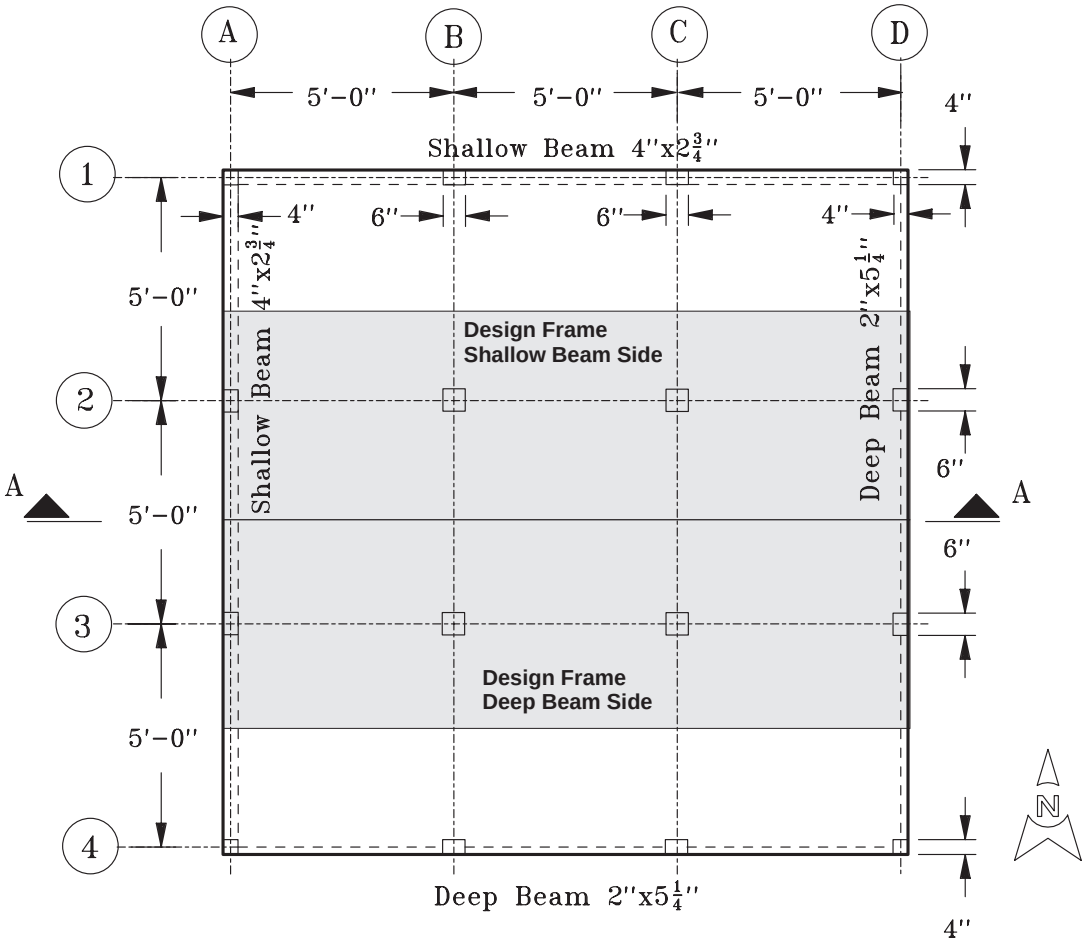
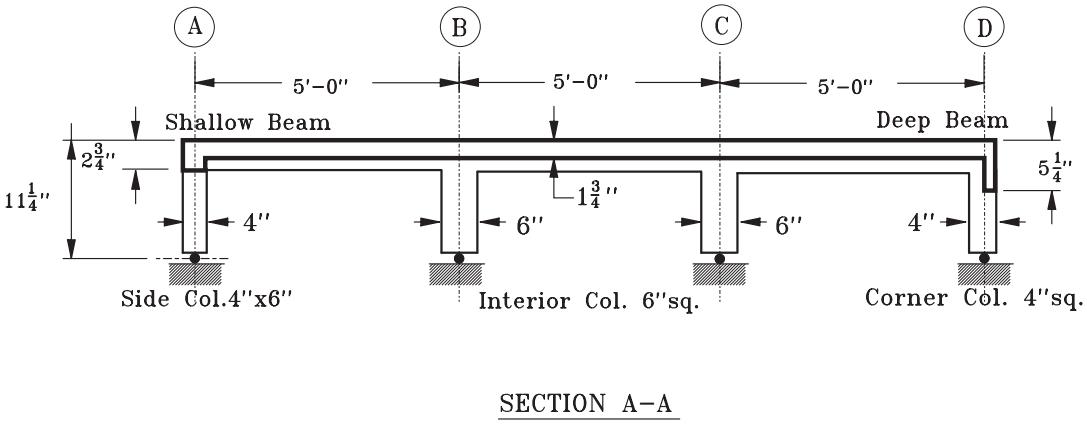


Figure 11-1
University of Illinois Flat Plate Test F1



SECTION A–A

Figure 11-2
Sections and Details of University of Illinois Flat Plate Test F1

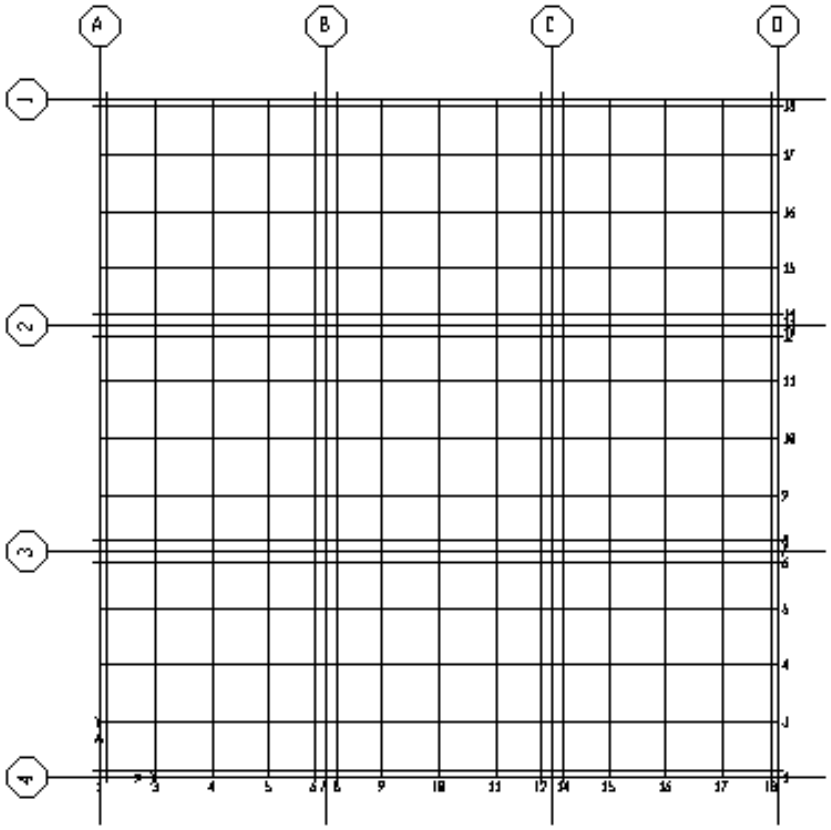


Figure 11-3
SAFE Mesh (6 × 6 per panel)

Method	Moments in an Interior Design Frame (M/Wl_1 *)								
	End Span (Shallow Beam Side)			Middle Span			End Span Deep Beam Side		
	^{-}M	^{+}M	^{-}M	^{-}M	^{+}M	^{-}M	^{-}M	^{+}M	^{-}M
TEST F1	0.027	0.049	0.065	0.064	0.040	0.058	0.058	0.047	0.034
EFM	0.047	0.044	0.072	0.066	0.034	0.067	0.073	0.044	0.046
SAFE (Shallow Beam Side)	0.044	0.050	0.068	0.062	0.041	0.062	0.067	0.051	0.042
SAFE (Deep Beam Side)	0.044	0.050	0.067	0.062	0.041	0.061	0.066	0.050	0.042

* $Wl_1 = 17.5$ *kip-ft*

Table 11-1
Comparison of Measured and Computed Moments

Example 12

University of Illinois Flat Slab Tests F2 and F3

Description

This example models, F2 and F3, the flat slab structures tested by the University of Illinois by Hatcher, Sozen, and Siess (1969) and Jirsa, Sozen, and Siess (1966) respectively. A typical structure used in tests F2 and F3 is shown in Figure 12-1. The fundamental difference between these two test structures is in the type of reinforcement used. In test F2, the slab is reinforced with medium grade reinforcement whereas in test F3 welded wire fabrics are used. The structure consists of nine $5' \times 5'$ panels arranged three-by-three. Two adjacent edges are supported by deep beams, 2 in. wide by 6 in. deep, and the other two edges by shallow beams, 4.5 in. wide by 2.5 in. deep, producing a single diagonal line of symmetry through columns A1, B2, C3, and D4. A typical section and details of columns, drop panels, and column capitals are shown in Figure 12-2. For both structures, the numerical results obtained for an interior frame by SAFE are compared with the experimental results and the EFM results due to uniformly distributed load.

Data

Concrete strength:

$f_c' = 2.76$ ksi (Test F2)
$f_c' = 3.76$ ksi (Test F3)

Yield strength of slab reinforcement:

f_y = 49 ksi (Test F2)
f_y = 54 ksi (Test F3)

Modulus of elasticity:

E_c = 2100 ksi (Test F2)
E_c = 3700 ksi (Test F3)

Poisson's ratio:

ν = 0.2

Loading:

Total uniform design load, w = 280 psf

Modeling Procedure

The computational model uses a 8×8 mesh of elements per panel as shown in Figure 12-3. The mesh contains gridlines at the column centerlines as well as the edges of drop panels and interior column capitals. The mesh extends to the centerlines of the edge columns only. In the absence of edge beams, it is recommended to include an extra line of elements beyond the edge column centerlines up to the slab edge. The slab thickness is increased to 2.5 inches over the drop panels. A thickness of 4.5 inches is used to approximately model the interior capitals. Short deep beams are used to model the edge column capitals. In this model, the slab is modeled with thin plate elements and the columns are modeled as point supports with vertical and rotational stiffnesses. A stiffness coefficient of 4.91 ($K_c = 4.91 EI_c / l_c$) is used in the calculation of the support flexural stiffness based on a column height of 21.375 inches, measured from the mid-depth of the slab to the support center. Due to the presence of capitals, columns are treated as non-prismatic. Shallow and deep beams are defined on the edges with torsional constants of 15.4 in^4 and 17.8 in^4 respectively as described in Section 13.0 of ACI 318-95.

Both the test problems are modeled in SAFE with concrete modulus of elasticity of 2100 ksi. This affects the slab, beam, and column stiffnesses. Since the distribution of moment depends on the relative stiffnesses, the test problems are not modeled twice, one for $E_c = 2100$ ksi and the other for $E_c = 3700$ ksi.

The model is analyzed for uniform load. To obtain maximum factored moments in an interior design frame, the slab is divided into two interior and two exterior design

frames spanning in the X direction (E-W direction). Because of symmetry, results are shown for X-strips only. An interior design frame consists of one column strip and half of each adjacent middle strip.

Comparison of Results

The SAFE results for moments in an interior frame, are compared with the experimental and EFM results for both structures F2 and F3 in Table 12-1. The experimental and EFM results are all obtained from Corley and Jirsa (1970). Moments are compared at the edge of column capitals. Table 12-1 shows that the SAFE and the EFM results are in excellent agreement. In general, the measured positive moments appear to be lower than the SAFE and EFM values.

File Reference

The file for this example is S12.FDB which is included in the SAFE package.

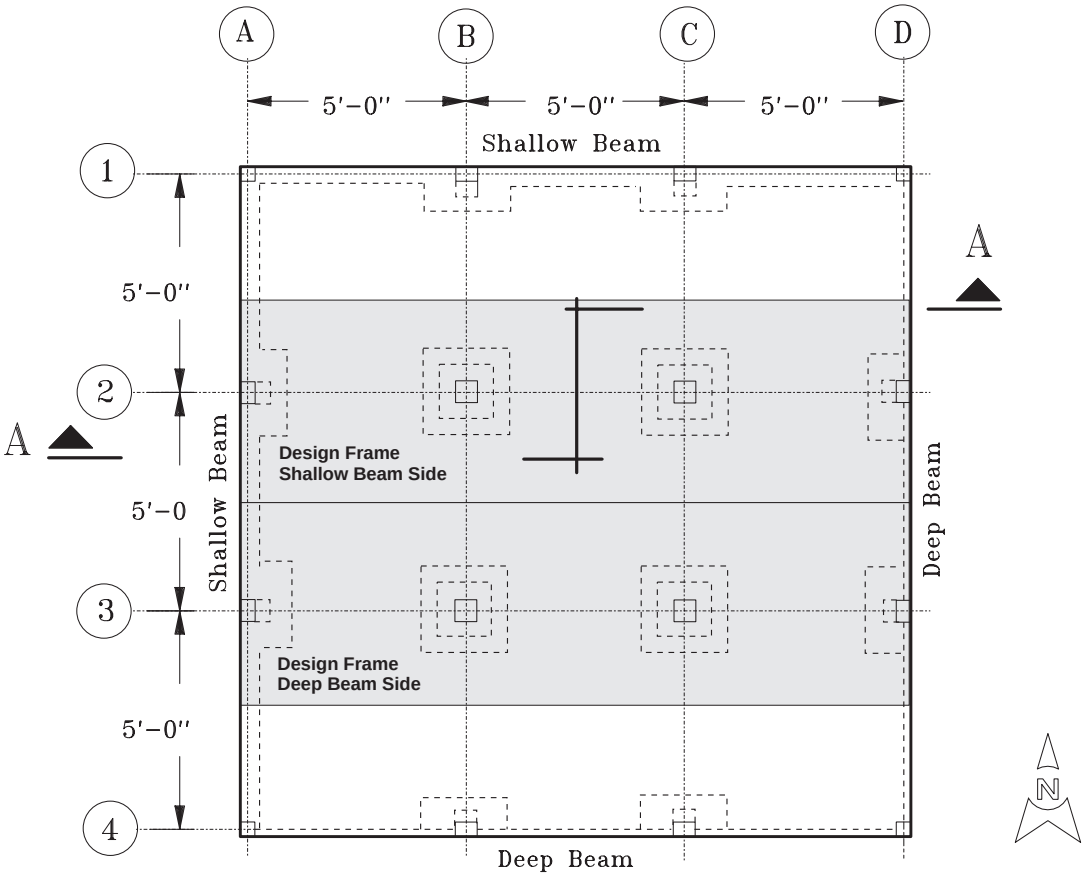


Figure 12-1
University of Illinois Flat Slab Tests F2 and F3

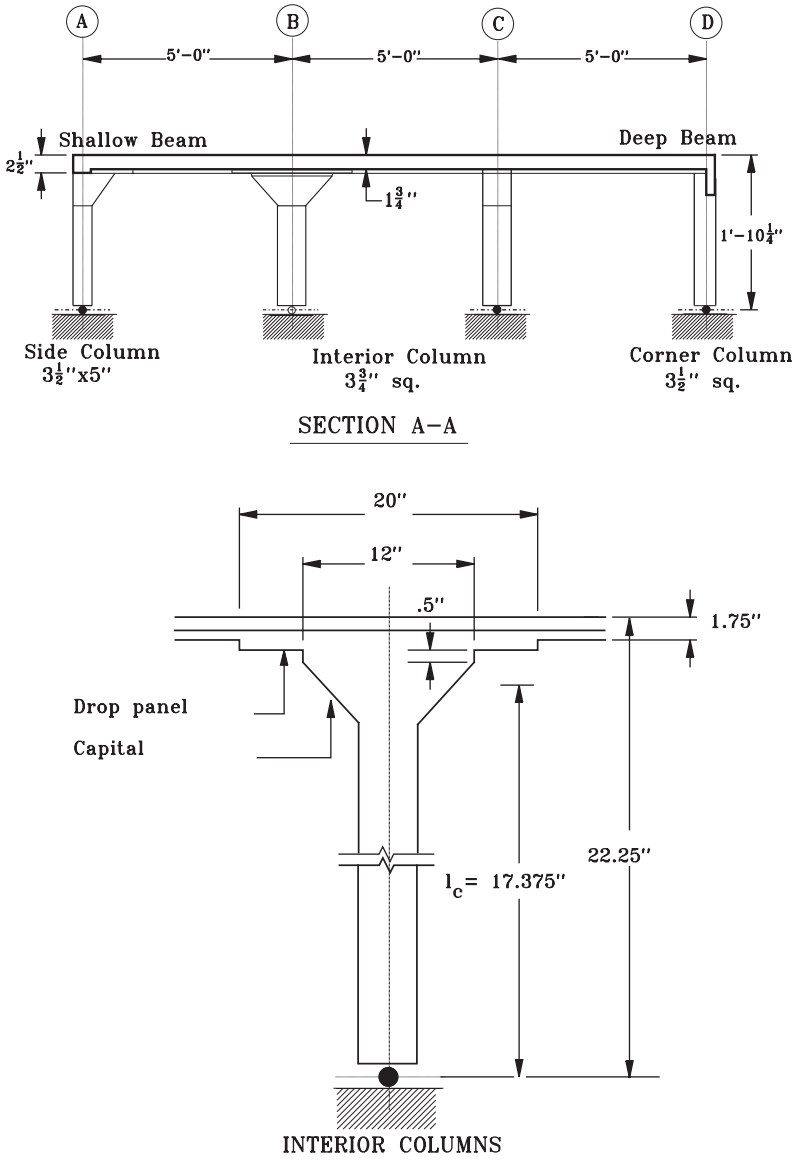


Figure 12-2
Sections and Details of Flat Slabs F2 and F3

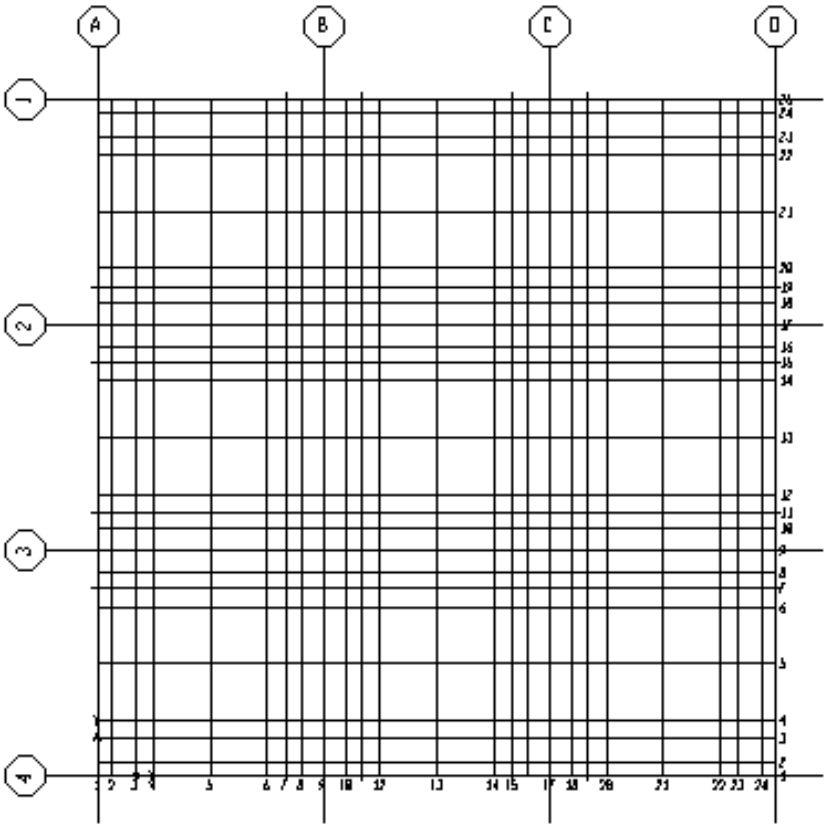


Figure 12-3
SAFE Mesh (8 × 8 per mesh)

Method	Moments in an Interior Design Frame (M/Wl_1 *)								
	End Span (Shallow Beam Side)			Middle Span			End Span (Deep Beam Side)		
	^{-}M	^{+}M	^{-}M	^{-}M	^{+}M	^{-}M	^{-}M	^{+}M	^{-}M
TEST F2	0.025	0.042	0.068	0.062	0.029	0.061	0.065	0.038	0.025
TEST F3	0.029	0.038	0.057	0.055	0.023	0.058	0.060	0.034	0.024
EFM	0.021	0.044	0.057	0.050	0.026	0.049	0.057	0.044	0.021
SAFE (Shallow Beam Side)	0.028	0.044	0.062	0.056	0.029	0.055	0.061	0.045	0.026
SAFE (Deep Beam Side)	0.028	0.043	0.062	0.055	0.029	0.055	0.061	0.044	0.026

* $Wl_1 = 35.0$ *kip-ft*

Table 12-1
Comparison of Measured and Computed Moments

Example 13

University of Illinois Two-way Slab Test T1

Description

This example models the slab structure tested by the University of Illinois by Gamble, Sozen, and Siess (1969). The structure is a two-way slab, 1.5 in. thick, in which each panel is supported along all four edges by beams as shown in Figure 13-1. The structure consists of nine $5' \times 5'$ panels arranged three-by-three. The edge beams extend $2.75''$ below the soffit of the slab and the interior beams have an overall depth of $5''$. The corner columns are $4'' \times 4''$ and the interior columns are $6'' \times 6''$. Edge columns are $4'' \times 6''$ with the longer dimension parallel to the slab edge. A typical section of the slab and details are shown in Figure 13-2. The moments in an interior design frame due to uniform loads obtained from SAFE are compared with the corresponding experimental results and the numerical values obtained from the EFM.

Data

Concrete strength,	f_c'	=	3	ksi
Yield strength of reinforcements,	f_y	=	42	ksi
Modulus of elasticity,	E_c	=	3000	ksi
Poisson's ratio,	ν	=	0.2	
Loading: Total uniform load	w	=	150	psf

Modeling Procedure

The computational model uses a 6×6 mesh of elements per panel as shown in Figure 13-3. Gridlines are defined at column faces as well as the column centerlines. The mesh extends to the edge column centerlines only. The slab is modeled using the thin plate elements available in SAFE. The columns are modeled as supports with both vertical and rotational stiffnesses. A stiffness coefficient of 8.0 is used in the calculation of support flexural stiffnesses based on a column height of $15.875''$, measured from the mid-depth of the slab to the support center. The column is assumed to be infinitely rigid over the full depth of the beams framing into it. The value of 8.0 is 75% of the figure obtained from Table 6.2 of ACI Committee 340 (1997) to approximately account for the pinned end condition at the column base. The slab is also thickened over the column sections to approximately model rigidity of the slab-column joint. Torsional constants of 33.7 in^4 and 24.2 in^4 are defined for the interior and edge beams respectively, in accordance with Section 13.0 of ACI 318-95.

To obtain maximum factored moments in an interior design frame, the slab is divided into two interior and two exterior design frames spanning in the X-direction (E-W direction). Because of double symmetry, comparison is confined to X-strips only. An interior design frame consists of one column strip and half of each adjacent middle strip.

Comparison of Results

The SAFE results for moments in an interior frame are compared with experimental and EFM results in Table 13-1. The test and EFM results are all obtained from Corley and Jirsa (1970). The negative design moments reported are at the face of columns. The comparison is excellent. The minor discrepancy is attributed to the loss of stiffness due to the development of cracks and the difficulty in measuring strains accurately at desired locations.

File Reference

The file for this example is S13.FDB which is included in the SAFE package.

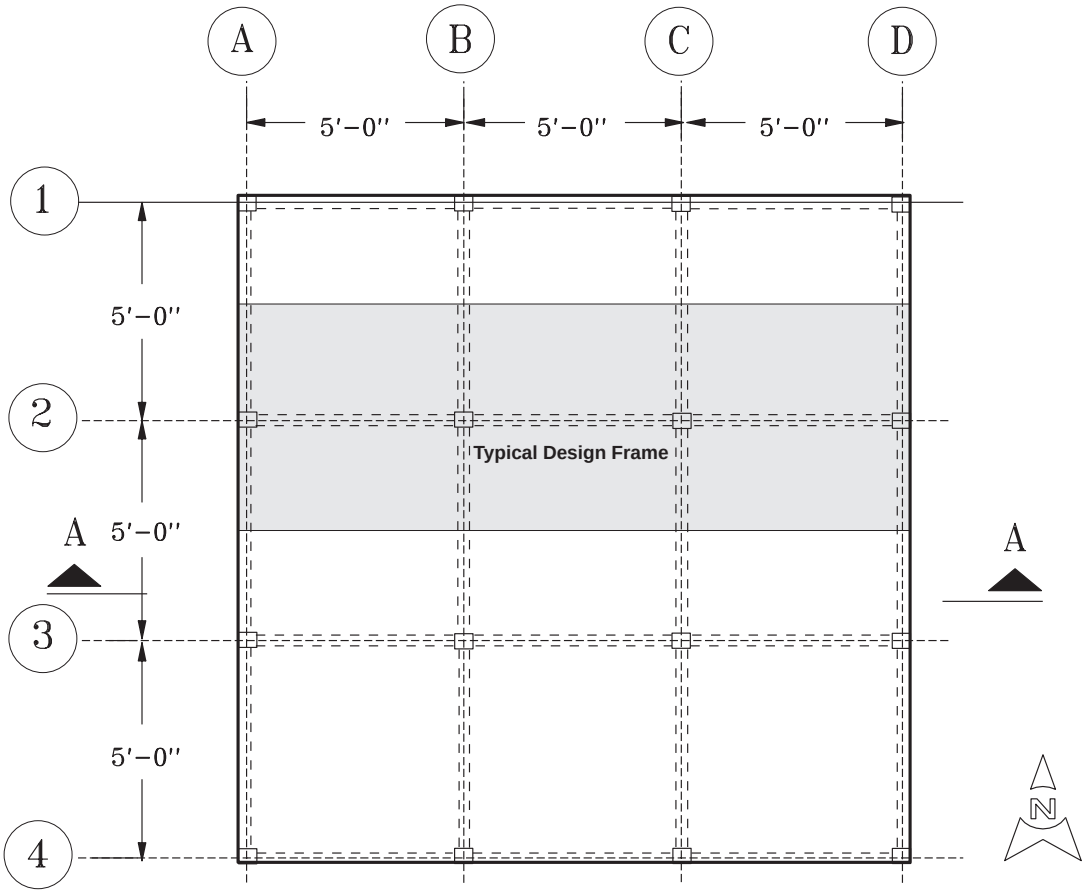


Figure 13-1
University of Illinois Two-way Slab Example T1

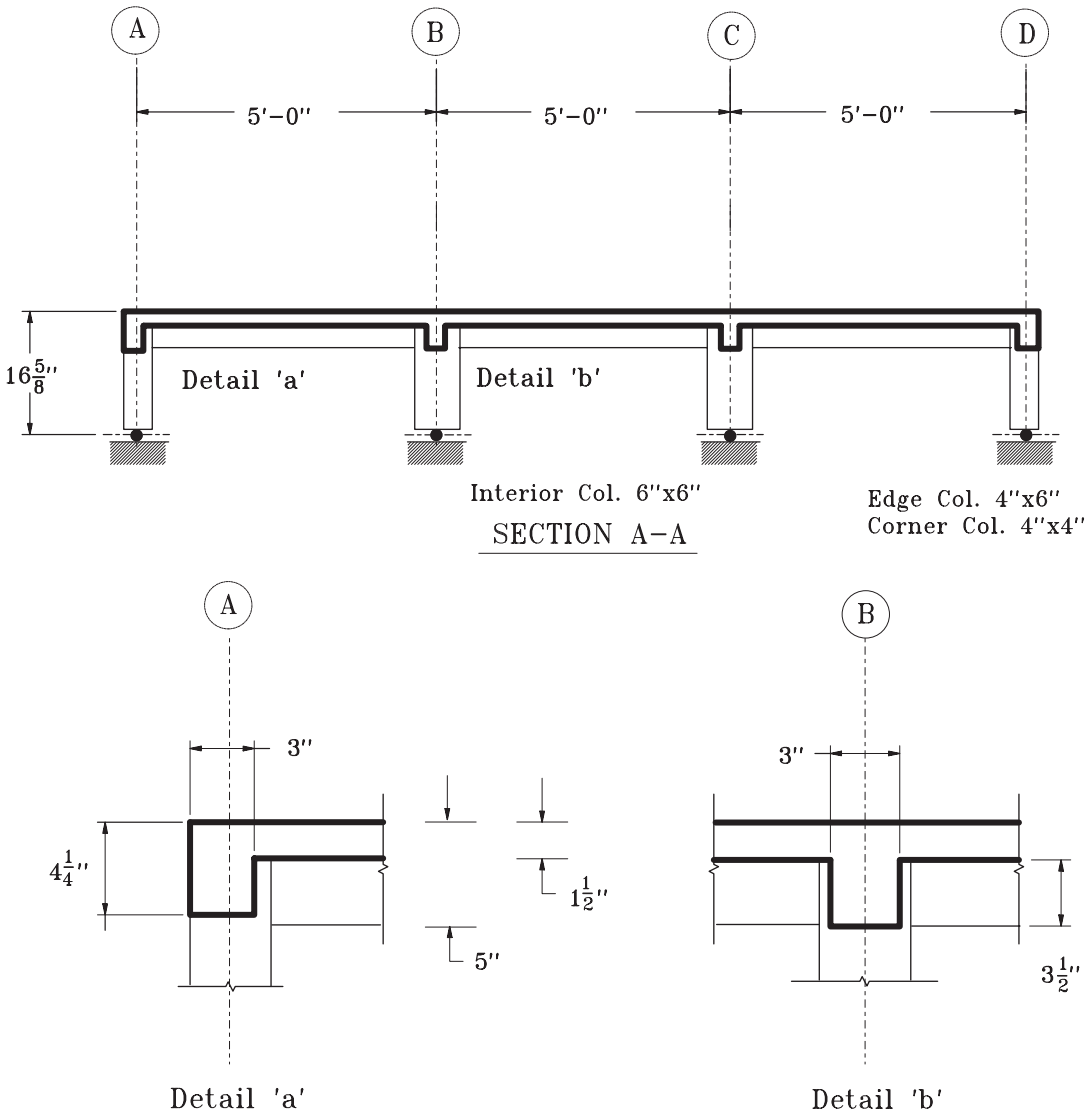


Figure 13-2
Sections and Details of Slab T1

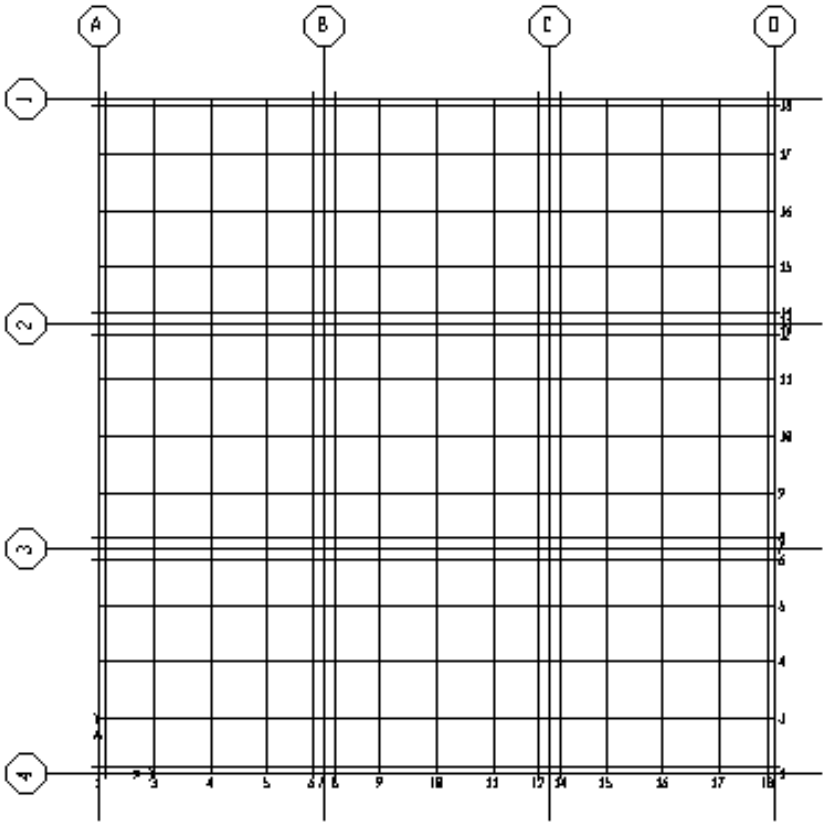


Figure 13-3
SAFE Mesh of Slab T1 (6 × 6 per panel)

Method	Moments in an Interior Design Frame (M/Wl_1 *)					
	Exterior Span			Middle Span		
	^{-}M	^{+}M	^{-}M	^{-}M	^{+}M	^{-}M
TEST T1	0.043	0.046	0.079	0.071	0.036	0.071
EFM	0.035	0.047	0.079	0.066	0.034	0.066
SAFE	0.043	0.049	0.074	0.062	0.041	0.062

* $Wl_1 = 18.75$ *kip-ft*

Table 13-1
Comparison of Measured and Computed Moments

Example 14

University of Illinois Two-way Slab Test T2

Description

This example models the slab structure tested by the University of Illinois by Vanderbilt, Sozen, and Siess (1969). The structure is a two-way slab arranged three-by-three panels in which each panel is supported along all four edges by beams as shown in Figure 14-1. The structure consists of nine $1.5^{\prime\prime}$ thick $5^{\prime} \times 5^{\prime}$ panels. The edge beams and the interior beams extend $1.5^{\prime\prime}$ below the soffit of the slab. The corner columns are $4^{\prime\prime} \times 4^{\prime\prime}$ and the interior columns are $6^{\prime\prime} \times 6^{\prime\prime}$. Edge columns are $4^{\prime\prime} \times 6^{\prime\prime}$ with the longer dimension parallel to the slab edge. A typical section of the slab and details are shown in Figure 14-2. The moments in an interior design frame obtained numerically from SAFE are compared with the experimental results and the EFM results.

Data

Concrete strength,	$f_c^{\prime}$	=	3	ksi
Yield strength of reinforcement,	f_y	=	47.6	ksi
Modulus of elasticity,	E_c	=	3000	ksi
Poisson's ratio,	ν	=	0.2	
Loading: Total uniform load,	w	=	139	psf

Modeling Procedure

The computational model uses a 6×6 mesh of elements per panel as shown in Figure 14-3. Gridlines are defined at column faces as well as the column centerlines. The mesh extends to the edge column centerlines only. The slab is modeled with thin plate elements and the columns are modeled as supports with both vertical and rotational stiffnesses. A stiffness coefficient of 6.33 is used in the calculation of support flexural stiffnesses based on a column height of 13.125 in., measured from the mid-depth of the slab to the support center. The column stiffness is assumed to be infinitely rigid over the full depth of the beams framing into it. The value of 6.33 is 75% of the figure obtained from Table A7 of Portland Cement Association (1990) to approximately account for the pinned end condition at the column base. The slab is also thickened over the column section to allow for rigidity of the slab-column joint. Torsional constants of 11.2 in^4 and 10.6 in^4 are defined for the interior and edge beams respectively, in accordance with Section 13.0 of ACI 318-95.

To obtain maximum factored moments in an interior design frame, the slab is divided into two interior and two exterior design frames spanning in the X-direction (E-W direction). An interior design frame consists of one column strip and half of each adjacent middle strip.

Comparison of Results

The SAFE results for moments in an interior frame are compared with the experimental and EFM results in Table 14-1. The experimental and EFM results are all obtained from Corley and Jirsa (1970). The negative design moments reported are at the face of columns. The comparison is excellent except for the negative exterior moments where the experimental results are lower than both the SAFE and the EFM results. The discrepancy is attributed not only to the loss of stiffness due to the development of cracks, but also to the difficulty in taking accurate strain measurements at desired locations.

File Reference

The file for this example is S14.FDB which is included in the SAFE package.

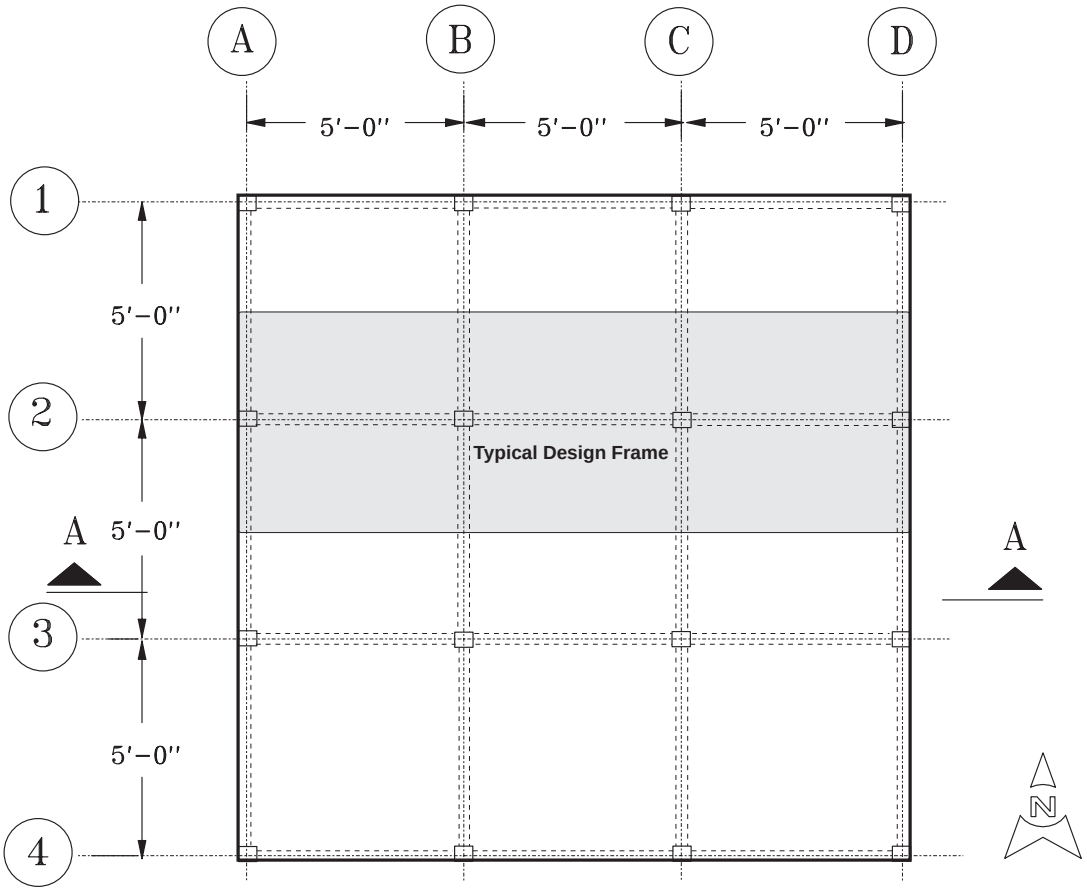


Figure 14-1
University of Illinois Two-Way Floor Slab T2

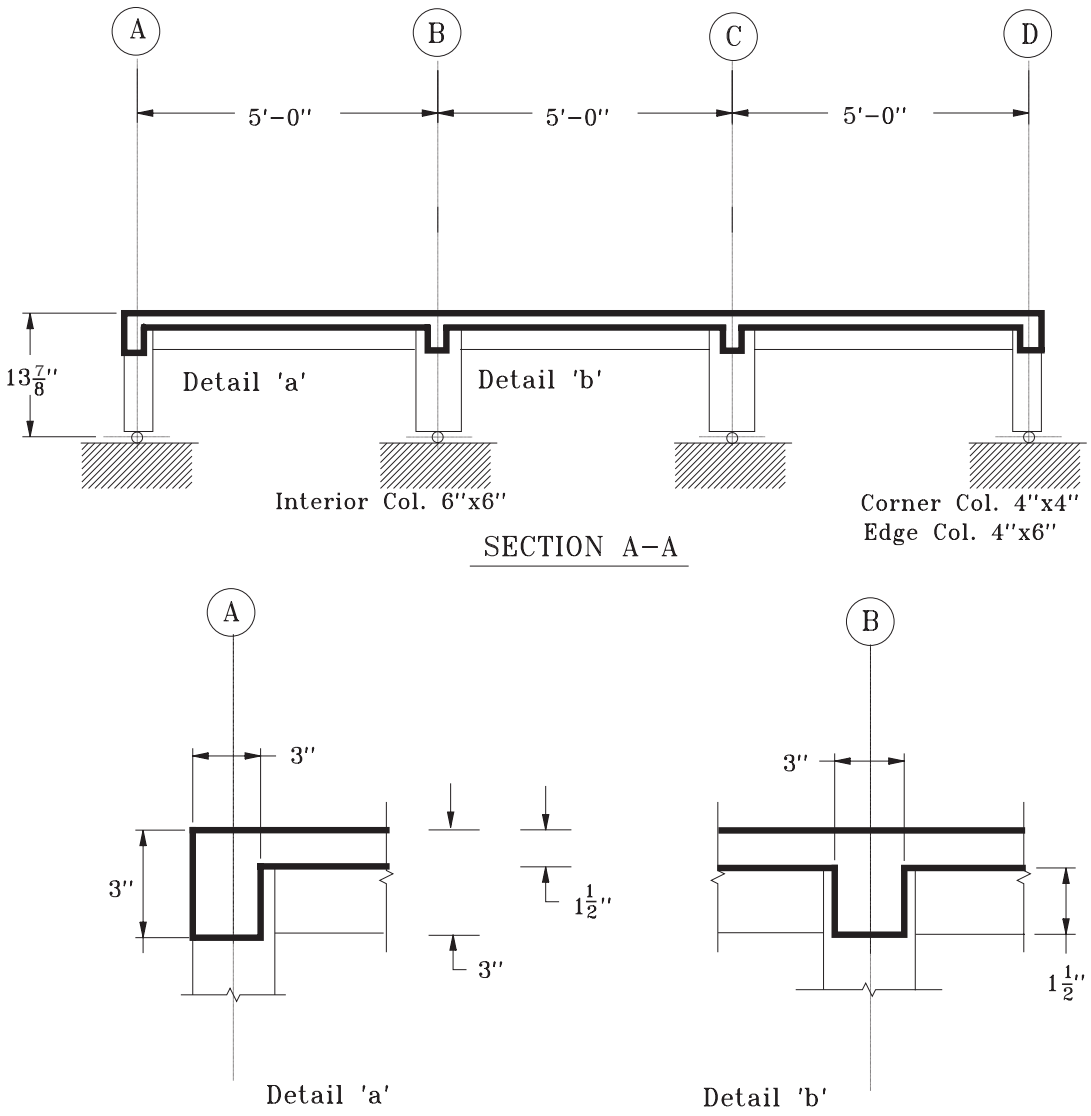


Figure 14-2
Sections and Details of Floor Slab T2

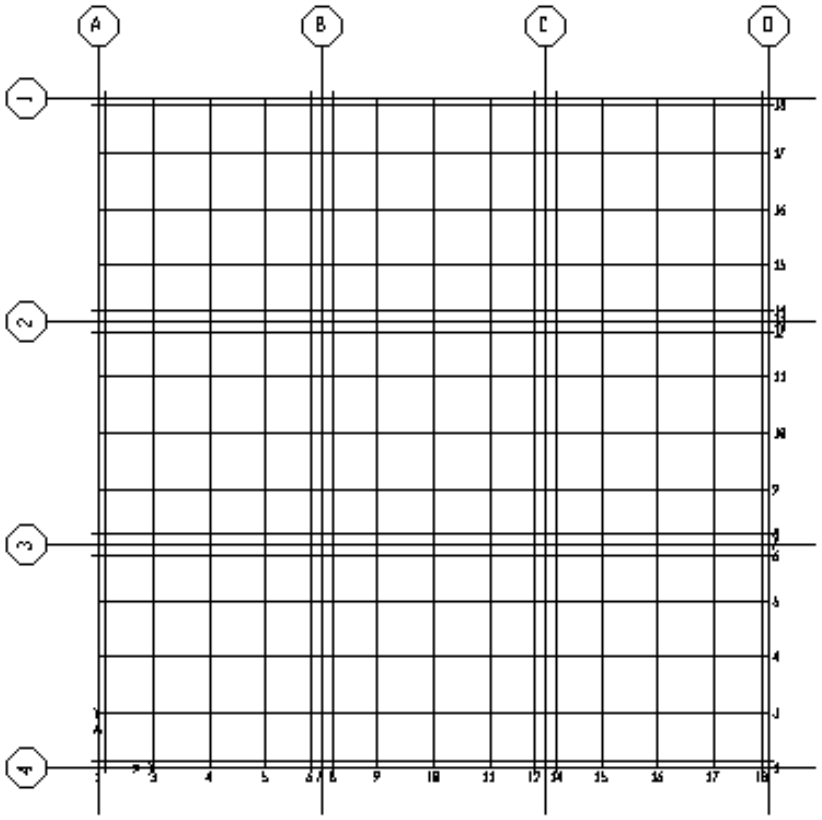


Figure 14-3
SAFE Mesh of Slab T2 (6 × 6 per panel)

Method	Moments in an Interior Design Frame (M/Wl_1 *)					
	Exterior Span			Middle Span		
	^{-}M	^{+}M	^{-}M	^{-}M	^{+}M	^{-}M
TEST T1	0.036	0.056	0.069	0.061	0.045	0.061
EFM	0.046	0.044	0.074	0.066	0.034	0.066
SAFE	0.048	0.048	0.070	0.062	0.041	0.062

* $Wl_1 = 17.375$ *kip-ft*

Table 14-1
Comparison of Measured and Computed Moments

Example 15

Design Verification of Slab

Description

The purpose of this example is to verify slab flexural design in SAFE for different load levels. The load levels are adjusted for three different cases corresponding to the following conditions:

- The computed tensile reinforcement falls below the minimum permitted in ACI,
- The computed tensile reinforcement falls above the minimum permitted in ACI but below the balanced condition,
- The computed tensile reinforcement exceeds the balanced condition.

A one-way simple-span slab supported by walls on both long edges is modeled using SAFE. The slab consists of a 6 inches thick 12 feet wide panel and is shown in Figure 15-1. To ensure one-way action Poisson's ratio is taken to be zero. The slab moment on a strip of unit width is computed analytically. The total factored strip moments are compared with the SAFE results. These moments are identical. After the analysis was done, design was performed using the ACI 318-95 code by SAFE and also by hand computation. The design reinforcements computed by the two methods are also compared in Table 15-1.

Data

Thickness,	T, h	= 6	in
Depth of tensile reinf.,	d_c	= 1	in
Effective depth,	d	= 5	in
Depth of comp. reinf.,	d'	= 1	in
Clear span,	l_n, l_1	= 144	in
Length,	l_2	= 720	in
Concrete strength,	$f_c^{'}$	= 4,000	psi
Yield strength of steel,	f_y	= 60,000	psi
Concrete unit weight,	w_c	= 0	pcf
Modulus of elasticity,	E_c	= 3,600	ksi
Modulus of elasticity,	E_s	= 29,000	ksi
Poisson's ratio,	ν	= 0	
Dead load,	w_d	= 80	psf
Live load,	w_l	= variable (0, 100, 800 psf)	

Modeling Procedure

The computational model uses a finite element mesh, automatically generated by SAFE. The maximum element size was specified to be 48 inches. To obtain factored moments and flexural reinforcement in a design strip, one one-foot wide strip is defined in X-direction on the slab as shown in Figure 15-1. The slab is modeled with thin plate elements. The walls are modeled as line supports without rotational stiffnesses and with very large vertical stiffness (1×10^{20} Kip/in.).

One dead load (DL80) and three live load (LL000, LL100, LL800) cases with uniformly distributed surface loads of magnitudes 80, 0, 100, and 800 psf, respectively, are defined in the model. Three load combinations (COMB000, COMB100, and COMB800) are defined with the ACI 318-95 load combination factors, 1.4 for dead loads and 1.7 for live loads. The model is analyzed for these load cases and load combinations.

Calculation of Reinforcement

The following quantities are computed for all the load combinations:

$\varphi = 0.90$

$b = 12$ in

$A_{s,min} = 0.0018\,bh = 0.1296$ sq-in

$\beta_1 = 0.85 - 0.05\left(\frac{f_c' - 4000}{1000}\right) = 0.85$

$c_b = \frac{87\,000}{87\,000 + f_y}\,d = 2.959$ in

$a_{max} = 0.75\beta_1 c_b = 1.8865$ in

For each load combination, the w and M_u are calculated as follows:

$w = (1.4w_d + 1.7w_l)\ b / 144$

$M_u = \frac{wl_1^2}{8}$

Low Load Level (COMB000)

$w_d = 80$ psf

$w_l = 0$ psf

$w = 9.333$ lb/in

$M_u = 24.192$ kip-in

The depth of the compression block is given by:

$a = d - \sqrt{d^2 - \frac{2\,|M_u|}{0.85 f_c' \varphi\, b}} = 0.1335 \text{ in} < a_{max}$

The area of tensile steel reinforcement is then given by:

$A_s = \frac{M_u}{\varphi f_y\left(d - \frac{a}{2}\right)} = 0.0908 \text{ sq-in} < A_{s,min}$

$A_s = \min\left[A_{s,min}, (4/3)A_{s,required}\right] = \min[0.1296, (4/3)\,0.0908] = 0.1296$ sq-in

Medium Load Level (COMB100):

w_d = 80 psf

w_l = 100 psf

w = 23.5 lb/in

M_u = 60.912 kip-in

The depth of the compression block is given by:

$$a = d - \sqrt{d^2 - \frac{2|M_u|}{0.85 f_c' \varphi b}} = 0.3436 \text{ in} < a_{max}$$

The area of tensile steel reinforcement is then given by:

$$A_s = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2} \right)} = 0.2336 \text{ sq-in} > A_{s,min}$$

A_s = 0.2336 sq-in

High Load Level (COMB800)

w_d = 80 psf

w_l = 800 psf

w = 122.67 lb/in

M_u = 317.952 kip-in

The depth of the compression block is given by:

$$a = d - \sqrt{d^2 - \frac{2|M_u|}{0.85 f_c' \varphi b}} = 2.2283 \text{ in} > a_{max}$$

The compressive force developed in concrete alone is given by:

$C = 0.85 f_c' b a_{max}$ = 76.968 kip

The moment resisted by concrete compression and tensile steel is:

$$M_{uc} = C\left(d - \frac{a_{max}}{2}\right)\varphi = \text{ 281.018 kip-in}$$

Therefore the moment resisted by compression steel and tensile steel is:

$$M_{us} = M_u - M_{uc} = 36.934 \text{ kip-in}$$

The stress in the compression steel is given by:

$$f_s' = 0.003 E_s \left[\frac{a_{max}/\beta_1 - d'}{a_{max}/\beta_1}\right] = 47.8 \text{ ksi} < f_y$$

So the area of required compression steel is given by

$$A_s' = \frac{M_{us}}{(f_s' - 0.85 f_c')(d - d')\varphi} = 0.2311 \text{ sq-in}$$

The required tensile steel for balancing the compression in concrete is:

$$A_{s1} = \frac{M_{uc}}{f_y(d - \frac{a_{max}}{2})\varphi} = 1.2828 \text{ sq-in}$$

The tensile steel for balancing the compression in steel is given by:

$$A_{s2} = \frac{M_{us}}{f_y(d - d')\varphi} = 0.1710 \text{ sq-in}$$

The total area of tensile steel is given by:

$$A_s = A_{s1} + A_{s2} = 1.4538 \text{ sq-in}$$

Comparison of Results

The SAFE total factored moments in the design strip are compared with the moments obtained by the analytical method in Table 15-1. They match exactly for this problem. The design reinforcements are also compared in Table 15-1.

File Reference

The file for this example is S15.FDB which is included in the SAFE package.

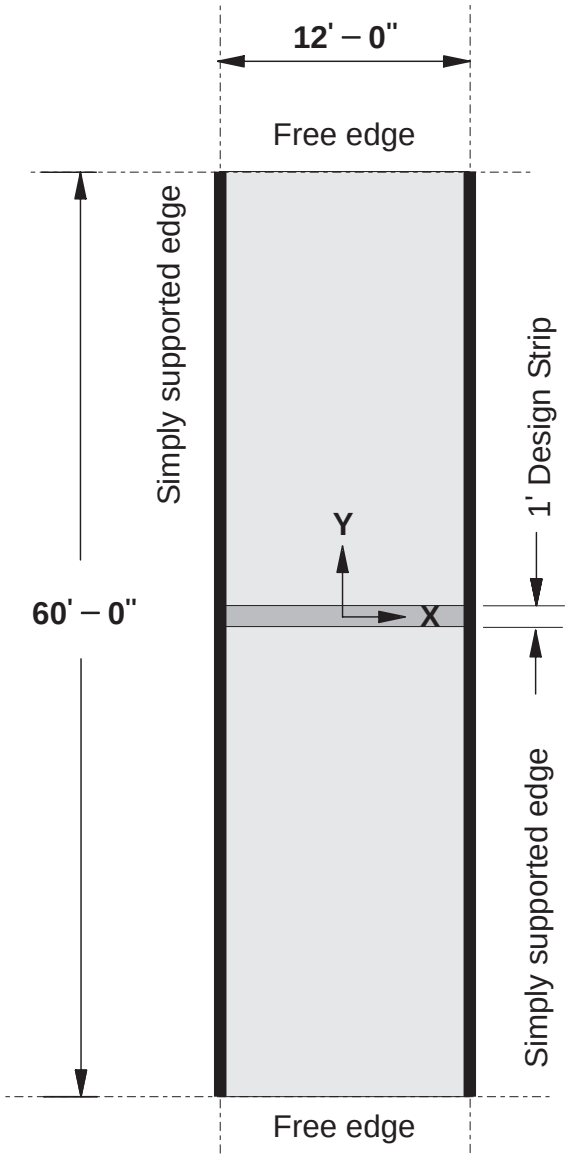


Figure 15-1
One-way Slab

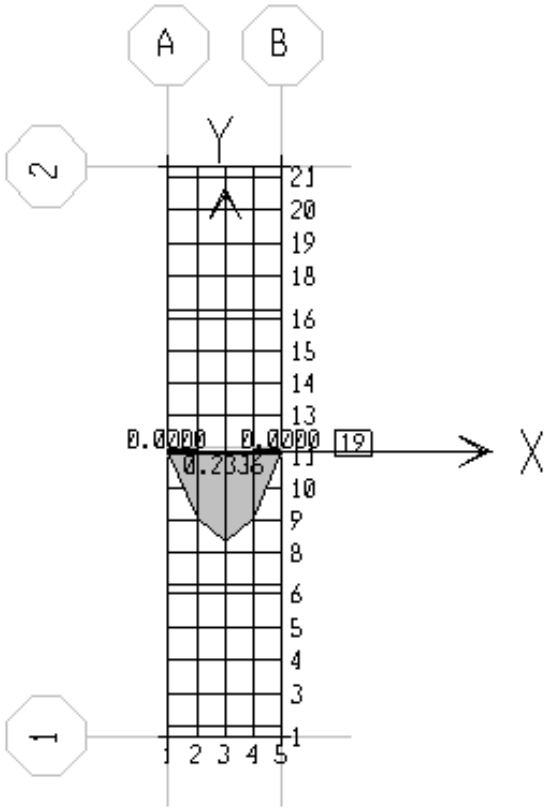


Figure 15-2
Strip Reinforcement for Medium Load Levels

Load Level	Method	Moment (kip-in)	Reinforcement Area (sq-in)	
			A_s^+	A_s^-
Low	SAFE	24.912	0.1296 *	—
	Calculated	24.912	0.1296 *	—
Medium	SAFE	60.912	0.2336	—
	Calculated	60.912	0.2336	—
High	SAFE	317.952	1.4538	0.2311
	Calculated	317.952	1.4538	0.2311

* $A^+_{s,min} = 0.1296$ *sq-in*

Table 15-1
Comparison of Design Moments and Reinforcements

Example 16

Flexural Design Verification of Beam

Description

The purpose of this example is to verify beam flexural design in SAFE for different load levels. A T-Beam section is considered. The load levels are adjusted for four different cases corresponding to the following conditions:

- The stress-block remains within the flange and the computed tensile reinforcement falls below the minimum permitted in ACI,
- The stress-block remains within the flange, the computed tensile reinforcement exceeds the minimum permitted in ACI, and remains within the balanced condition permitted by ACI,
- The stress-block extends below the flange but remains within the balanced condition permitted by ACI,
- The stress-block extends below the flange and exceeds the permitted balanced condition, requiring compression reinforcement.

A simple-span $20'$ long, $12''$ wide, and $18''$ deep T-beam with a flange of $4''$ thickness and $24''$ width is modeled using SAFE. The beam is shown in Figure 16-1. The beam is loaded with symmetric third-point loading. The beam moment can be computed analytically. The total factored moments are compared with the SAFE results. They are identical. After the analysis was done, design was performed using the ACI 318-95 code in SAFE and by hand computation. The design longitudinal reinforcements are compared in Table 16-1.

Data

Clear span,	l	= 240	in
Overall depth,	h	= 18	in
Flange Thickness,	d_s	= 4	in
Width of web,	b_w	= 12	in
Width of flange,	b_f	= 24	in
Depth of tensile reinf.,	d_c	= 3	in
Effective depth,	d	= 15	in
Depth of comp. reinf.,	d'	= 3	in
Concrete strength,	f'_c	= 4,000	psi
Yield strength of steel,	f_y	= 60,000	psi
Concrete unit weight,	w_c	= 0	pcf
Modulus of elasticity,	E_c	= 3,600	ksi
Modulus of elasticity,	E_s	= 29,000	ksi
Poisson's ratio,	ν	= 0.2	
Dead load,	P_d	= 3	kips
Live load,	P_l	= variable (0, 10, 30, 40 kips)	

Modeling Procedure

The computational model uses a finite element mesh of frame elements, automatically generated by SAFE. The maximum element size was specified to be 6 inches. The beam is supported by columns without rotational stiffnesses and with very large vertical stiffness (1×10^{20} Kip/in).

One dead load (DL03) and four live load (LL00, LL10, LL30, LL40) cases with only symmetric third-point loads of magnitudes 3, 0, 10, 30, and 40 kips, respectively, are defined in the model. Three load combinations (COMB00, COMB10, COMB30, and COMB40) are defined with the ACI 318-95 load combination factors of 1.4 for dead loads and 1.7 for live loads. The model is analyzed for all of these load cases and load combinations.

Calculation of Reinforcement

The following quantities are computed for all the load combinations (Please refer to the SAFE User's Manual for more details):

$\varphi = 0.90$

$$A_s \geq \max\left\{ \frac{3\sqrt{f_c'}}{f_y} b_w d \quad \text{and} \quad \frac{200}{f_y} b_w d \right\} = 0.60 \text{ sq-in}$$

$$\beta_1 = 0.85 - 0.05\left(\frac{f_c' - 4000}{1000} \right) = 0.85$$

$$c_b = \frac{87\,000}{87\,000 + f_y} d = 8.8776 \text{ in}$$

$$a_{max} = 0.75\beta_1 c_b = 5.6594 \text{ in}$$

For each load combination, the P_u and M_u are calculated as follows:

$$P_u = 1.4P_d + 1.7P_l$$

$$M_u = \frac{P_u L}{3}$$

Low Load Level (COMB00)

P_d = 3 kips

P_l = 0 kip

P_u = 4.2 kips

M_u = 336 kip-in

The depth of the compression block is given by:

$$a = d - \sqrt{d^2 - \frac{2 M_u}{0.85 f_c' \varphi b_f}} = 0.3082 \text{ in} \quad (a < a_{max} \text{ and } a < d_s)$$

The area of tensile steel reinforcement is then given by:

$$A_s = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2} \right)} = 0.4191 \text{ sq-in} < A_{s,min}$$

$$A_s = \min\left[A_{s,min}, (4/3) A_{s,required} \right] = \min[0.60, (4/3)\,0.4191] = 0.5588 \text{ sq-in}$$

Medium Load Level (COMB10):

P_d = 3 kips

P_l = 10 kips

P_u = 21.2 kips

M_u = 1696 kip-in

The depth of the compression block is given by:

$$a = d - \sqrt{d^2 - \frac{2 M_u}{0.85 f_c' \varphi b_f}} = 1.6279 \text{ in } (a < a_{max} \text{ and } a < d_s)$$

The area of tensile steel reinforcement is then given by:

$$A_s = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2} \right)} = 2.2140 \text{ sq-in} > A_{s,min}$$

A_s = 2.2140 sq-in

High Load Level (COMB30)

P_d = 3 kips

P_l = 30 kips

P_u = 55.2 kips

M_u = 4416 kip-in

The depth of the compression block is given by:

$$a = d - \sqrt{d^2 - \frac{2 M_u}{0.85 f_c' \varphi b_f}} = 4.7658 \text{ in } (a > d_s)$$

Calculation for A_s is done in two parts. The first part is for balancing the compressive force from the flange, C_f, and the second part is for balancing the compressive force from the web, C_w. C_f is given by:

$$C_f = 0.85 f_c' (b_f - b_w) d_s = 163.2 \text{ kips}$$

The portion of M_u that is resisted by the flange is given by:

$$M_{uf} = C_f \left(d - \frac{d_s}{2} \right) \varphi = 1909.44 \text{ kip-in}$$

Therefore, the area of tensile steel reinforcement to balance flange compression is:

$$A_{s1} = \frac{M_{uf}}{f_y (d - d_s/2) \varphi} = 2.7200 \text{ sq-in}$$

The balance of the moment to be carried by the web is given by:

$$M_{uw} = M_u - M_{uf} = 2506.56 \text{ kip-in}$$

The web is a rectangular section of dimensions b_w and d, for which the design depth of the compression block is recalculated as

$$a_1 = d - \sqrt{d^2 - \frac{2 M_{uw}}{0.85 f_c^{'} \varphi b_w}} = 5.5938 \text{ in} \quad (a_1 \le a_{max})$$

The area of tensile steel reinforcement to balance the web compression is then given by:

$$A_{s2} = \frac{M_{uw}}{\varphi f_y \left(d - \frac{a_1}{2} \right)} = 3.8037 \text{ sq-in}$$

The area of total tensile steel reinforcement is then given by:

$$A_s = A_{s1} + A_{s2} = 6.5237 \text{ sq-in}$$

Very High Load Level (COMB40)

P_d = 3 kips

P_l = 40 kips

P_u = 72.2 kips

M_u = 5776 kip-in

The depth of the compression block is given by:

$$a = d - \sqrt{d^2 - \frac{2 M_u}{0.85 f_c' \varphi b_f}} = 6.7719 \text{ in } (a > d_s)$$

Calculation for A_s is done in two parts. The first part is for balancing the compressive force from the flange, C_f, and the second part is for balancing the compressive force from the web, C_w. C_f is given by:

$$C_f = 0.85 f_c' (b_f - b_w) d_s = 163.2 \text{ kips}$$

The portion of M_u that is resisted by the flange is given by

$$M_{uf} = C_f \left(d - \frac{d_s}{2} \right) \varphi = 1909.44 \text{ kip-in}$$

Therefore, the area of tensile steel reinforcement to balance flange compression is:

$$A_{s1} = \frac{M_{uf}}{f_y (d - d_s/2) \varphi} = 2.7200 \text{ sq-in}$$

The balance of the moment to be carried by the web is given by:

$$M_{uw} = M_u - M_{uf} = 3866.56 \text{ kip-in}$$

The web is a rectangular section of dimensions b_w and d, for which the design depth of the compression block is recalculated as:

$$a_1 = d - \sqrt{d^2 - \frac{2 M_{uw}}{0.85 f_c' \varphi b_w}} = 11.2048 \quad (a_1 > a_{max})$$

Compression reinforcement is required. The compressive force in web concrete alone is given by:

$$C = 0.85 f_c' b a_{max} = 230.9051 \text{ kip-in}$$

Therefore the moment resisted by concrete web and tensile steel is:

$$M_{uc} = C \left(d - \frac{a_{max}}{2} \right) \varphi = 2529.1619 \text{ kip-in}$$

The moment resisted by compression steel and tensile steel is:

$$M_{us} = M_{uw} - M_{uc} = 1337.3981 \text{ kip-in}$$

The stress in the compression steel is given by:

$$f_s' = 0.003 E_s \left[\frac{a_{max}/\beta_1 - d'}{a_{max}/\beta_1} \right] = 47.8 \text{ ksi} < f_y$$

So the area of required compression steel is given by

$$A_s' = \frac{M_{us}}{(f_s' - 0.85 f_c')(d - d')\varphi} = 2.7890 \text{ sq-in}$$

The tensile steel for balancing compression in web concrete is:

$$A_{s2} = \frac{M_{uc}}{f_y (d - \frac{a_{max}}{2})\varphi} = 3.8484 \text{ sq-in}$$

The tensile steel for balancing compression in steel is:

$$A_{s3} = \frac{M_{us}}{f_y (d - d')\varphi} = 2.0639 \text{ sq-in}$$

The total tensile reinforcement is:

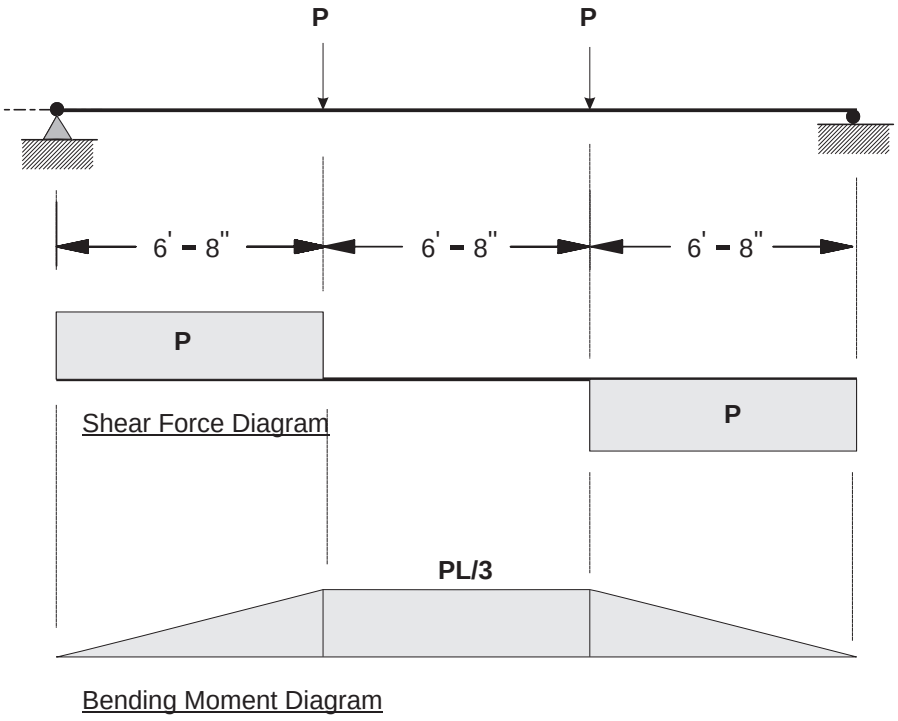
$$A_s = A_{s1} + A_{s2} + A_{s3} = 8.6323 \text{ sq-in}$$

Comparison of Results

The SAFE total factored moments in the beam for different load combinations are compared in with the moments obtained by the analytical method. They match exactly for this problem. The design reinforcements are also compared in Table 16-1. They also match exactly.

File Reference

The file for this example is S16.FDB which is included in the SAFE package.



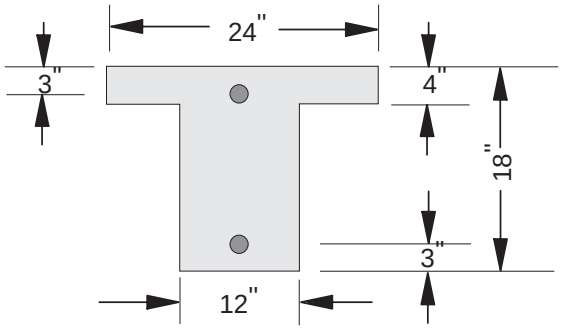


Figure 16-1
The Model Beam for Flexural Design

Load Level	Method	Moment (kip-in)	Reinforcement Area (sq-in)	
			A_s^+	A_s^-
Low	SAFE	336	0.5588	—
	Calculated	336	0.5588	—
Medium	SAFE	1696	2.2140	—
	Calculated	1696	2.2140	—
High	SAFE	4416	6.5237	—
	Calculated	4416	6.5237	—
Very High	SAFE	5776	O/S	2.7890
	Calculated	5776	8.6323 (O/S)	2.7890

Table 16-1
Comparison of Moments and Flexural Reinforcements

Example 17

Shear Design Verification of Beam

Description

The purpose of this example is to verify beam shear design according to the ACI 318-95 code in SAFE for different load levels. The load levels are adjusted for four different cases corresponding to the following conditions:

- The average shear stress in the beam falls below half of the concrete capacity, requiring no shear reinforcement,
- The average shear stress in the beam falls above half of the concrete capacity but not exceeding the concrete capacity by 50 psi, requiring minimum shear reinforcement according to the ACI 318-95 code,
- The average shear stress in the beam falls below the maximum shear stress allowed by ACI 318-95, requiring design shear reinforcement,
- The average shear stress in the beam exceeds the maximum shear force allowed by ACI 318-95 representing a failure condition.

A simple-span $20'$ long, $12''$ wide, and $18''$ deep T-beam with a flange of $4''$ thickness and $24''$ width is modeled using SAFE. The beam is shown in Figure 17-1. The beam is loaded with symmetric third-point loading. The beam shear force is computed analytically. The total factored shear forces are compared with the SAFE results. These shear forces are identical. After the analysis was done, design was performed using the ACI 318-95 code in SAFE and also by hand computation. The design shear reinforcements are compared in Table 17-1.

Data

Clear span,	l	= 240	in
Overall depth,	h	= 18	in
Flange Thickness,	h_f	= 4	in
Width of web,	b_w	= 12	in
Width of flange,	b_f	= 24	in
Depth of tensile reinf.,	d_c	= 3	in
Effective depth,	d	= 15	in
Depth of comp. reinf.,	d'	= 3	in
Concrete strength,	f'_c	= 4,000	psi
Yield strength of steel,	f_y	= 60,000	psi
Concrete unit weight,	w_c	= 0	pcf
Modulus of elasticity,	E_c	= 3,600	ksi
Modulus of elasticity,	E_s	= 29,000	ksi
Poisson's ratio,	ν	= 0.2	
Dead load,	P_d	= 3	kips
Live load,	P_l	= variable (0, 10, 30, 60 kips)	

Modeling Procedure

The computational model uses a finite element mesh of frame elements, automatically generated by SAFE. The maximum element size was specified to be 6 inches. The beam is supported by columns without rotational stiffnesses and with very large vertical stiffness (1×10^{20} Kip/in).

One dead load (DL03) and four live load (LL00, LL10, LL30, LL60) cases with only symmetric third-point loads of magnitudes 3, 0, 10, 30, and 60 kips, respectively, are defined in the model. Three load combinations (COMB00, COMB10, COMB30, and COMB60) are defined with the ACI 318-95 load combination factors of 1.4 for dead loads and 1.7 for live loads. The model is analyzed for all of these load cases and load combinations.

Calculation of Reinforcement

The following quantities are computed for all the load combinations (Please refer to the SAFE User's Manual for more details):

$\varphi = 0.85$

Check the limit of $\sqrt{f_c'}$:

$\sqrt{f_c'}$ = 63.246 psi < 100 psi

The concrete shear capacity is given by:

$\varphi V_c = \varphi\ 2\sqrt{f_c'}\ b_w d$ = 19.353 kips

The maximum shear that can be carried by reinforcement is given by:

$\varphi V_s = \varphi\ 8\sqrt{f_c'}\ b_w d$ = 77.41 kips

The following limits are required in the determination of the reinforcement:

$(\varphi V_c/2)$ = 9.677 kips

$(\varphi V_c + \varphi\ 50\, b_w d)$ = 27.003 kips

$V_{max} = \varphi V_c + \varphi V_s$ = 96.766 kips

Given V_u, V_c and V_{max}, the required shear reinforcement in area/unit length for any load combination is calculated as follows:

If $V_u \le (V_c/2)\varphi$,

$\frac{A_v}{s} = 0$,

else if $(V_c/2)\varphi < V_u \le (\varphi V_c + \varphi\ 50\, b_w d)$,

$\frac{A_v}{s} = \frac{50\ b_w}{f_y}$,

else if $(\varphi V_c + \varphi\ 50\, b_w d) < V_u \le \varphi V_{max}$,

$\frac{A_v}{s} = \frac{(V_u - \varphi V_c)}{\varphi f_{ys} d}$,

else if $V_u > \varphi V_{max}$,

a failure condition is declared.

For each load combination, the P_u and V_u are calculated as follows:

$$P_u = 1.4P_d + 1.7P_l$$

$$V_u = P_u$$

Low Load Level (COMB00)

P_d = 3 kips

P_l = 0 kip

P_u = 4.2 kips

V_u = 4.2 kip, $V_u \le (V_c/2)\varphi$

$\frac{A_v}{s}$ = 0

Medium Load Level (COMB10)

P_d = 3 kips

P_l = 10 kips

P_u = 21.2 kips

V_u = 21.2 kip, $(V_c/2)\varphi < V_u \le (\varphi V_c + \varphi\ 50\, b_w d)$

$\frac{A_v}{s} = \frac{50\ b_w}{f_y}$ = 0.01 sq-in/in or 0.12 sq-in/ft

High Load Level (COMB30)

P_d = 3 kips

P_l = 30 kips

P_u = 55.2 kips

V_u = 55.2 kip, $(\varphi V_c + \varphi\ 50\, b_w d) < V_u \le \varphi V_{max}$

$\frac{A_v}{s} = \frac{(V_u - \varphi V_c)}{\varphi f_{ys}\, d}$ = 0.04686 sq-in/in or 0.562 sq-in/ft

Very High Load Level (COMB60)

P_d = 3 kips

P_l = 60 kips

P_u = 106.2 kips

V_u = 106.2 kip, $V_u > \varphi V_{max}$

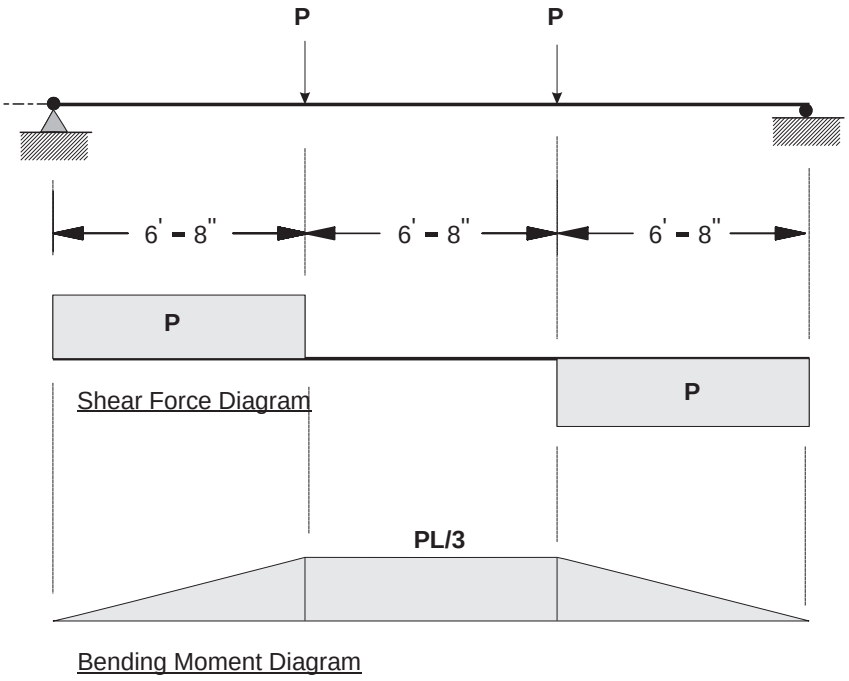
$\frac{A_v}{s} = \frac{(V_u - \varphi V_c)}{\varphi f_{ys} d}$ = 0.1135 sq-in/in, and a failure condition is declared.

Comparison of Results

The SAFE total factored shear-forces in the beam for different load combinations are compared in Table 17-1 with the shear-forces obtained by the analytical method. They match exactly for this problem. The design shear reinforcements are also compared in Table 17-1. They also match exactly.

File Reference

The file for this example is S17.FDB which is included in the SAFE package.



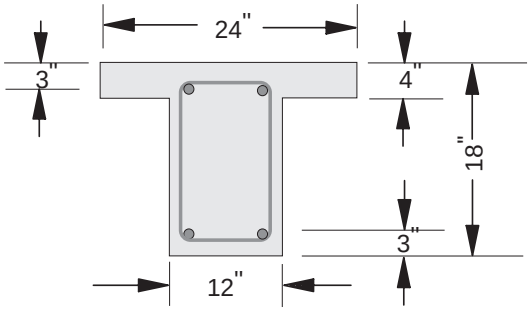


Figure 17-1
The Model Beam for Shear Design

Load Level	Shear Force (kip)	Reinforcement Area, $\frac{A_v}{s}$ (sq-in/ft)	
		SAFE	Calculated
Low	4.2	0.000	0.000
Medium	21.2	0.120	0.120
High	55.2	0.562	0.562
Very high	106.2	O/S	O/S

Table 17-1
Comparison of Shear Reinforcements

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